The Journal of Experimental Education, 64(1), 79–93

Multiple Regression Approach to Analyzing Contingency Tables: Post Hoc and Planned Comparison Procedures

T. MARK BEASLEY
St. John's University

RANDALL E. SCHUMACKER
University of North Texas

ABSTRACT. Post hoc and planned comparison procedures for interpreting chi-square contingency-table test results, not currently discussed in most standard textbooks, are presented. A planned comparison procedure that simplifies the tedious process of partitioning a contingency table by creating single-degree-of-freedom contrasts through a regression-based approach is proposed. Importantly, these post hoc methods supplement the analysis of standardized residuals by reporting the percentage contribution for each cell to the overall chi-square statistic (relative contribution) and to the percentage of variance shared by the two factors (absolute contribution). Both methods can be readily incorporated into existing statistical packages such as SAS or SPSS. The equivalence of the percentage contribution method to the more common standardized residual method is also presented along with an example of a typical application.

A COMMON APPLICATION of the Pearson chi-square test is when the frequencies in the different columns (or rows) of the contingency table "can be regarded as sampled from separate and distinct populations" (Glass & Stanley, 1970, p. 333). In this case, the null hypothesis is that the proportional distribution (π) of responses to a categorical variable are homogeneous (do not differ) across k samples,

$$H_0\text{: } \pi_1 = \pi_2 \ldots = \pi_k. \qquad (1)$$

It is also common to phrase the null hypothesis in terms of independence, in that the proportional makeup of the k samples is independent of another demographic attribute. For these analyses, an $R \times C$ contingency table (where R = the number of rows and C = the number of columns) is formed. For each of the $s = (R \times C)$ cells in the table, the difference between the observed frequency, f_{rc}, and the expected frequency, E_{rc}, is squared and divided by E_{rc}:

$$\chi^2 = \sum_{r=1}^{R} \sum_{c=1}^{C} \frac{(f_{rc} - E_{rc})^2}{E_{rc}}. \quad (2)$$

E_{rc} is derived by the multiplication of respective row and column marginal totals for each cell divided by the total sample size (sum of all frequencies), N. This test, under the null hypothesis, approximates a chi-square distribution with $v = (R - 1) \times (C - 1)$ degrees of freedom (*df*s).

Despite frequent use, however, chi-square tests are often misinterpreted. It is common for the chi-square test to be viewed as merely a nonparametric test of independent proportions. In a series of comments published approximately 40 years ago, Burke (1951) and Lewis and Burke (1949) noted limitations of contingency table analyses. The consensus based on these comments was that although the chi-square can be converted into a coefficient of contingency, it is "inadequate partly because it cannot reach 1.00, has no sign, and is not readily interpretable in various terms as is *r*" (Kerlinger, 1973, p. 172). As Best (1981) noted, "The chi square is not a measure of the degree of relationship," but is only a test of whether or not a null hypothesis of no association should be rejected (p. 287). That is, the chi-square statistic tests a general null hypothesis and does not inform the researcher as to the nature of the relationship between factors (Thompson, 1988). Furthermore, in most applications statistical significance is a direct function of sample size and, therefore, does not convey any practical information to a researcher (Thompson, 1994a). For this reason, other statistics are vital for the interpretation of findings, and, accordingly, no chi-square test should stop with the computation of an omnibus chi-square statistic. Clearly, a *percentage of shared variance* interpretation of chi-square results is needed.

Our focus in the present article is to demonstrate methods for analyzing contingency tables and identifying which cell(s) contribute the most to a statistically significant omnibus chi-square test. Our intent is to elaborate two methodologies that are not commonly found in standard statistics texts, but can be easily calculated, reported, and evaluated. Both approaches can be incorporated into existing statistical packages such as SAS or SPSS. We also discuss the equivalence of the proposed methodologies to more common methods and the omnibus chi-square test along with an example of a typical application.

Parametric Analogs to the Analysis of Contingency Tables

The chi-square test is more generally viewed as a test of association among cross-classified factors in a contingency table sampled from a single pool of subjects, and, therefore, it has equivalence to other distributions and has parametric analogs (Marascuilo & McSweeney, 1977). With $v = 1$, $\chi^2 = z^2 = (X - M)^2/S^2$, where M is the mean and S^2 is the variance of X. Thus, an explanation based on z values and the normal distribution is possible (Guilford & Fruchter, 1973, pp. 195–213). Because of its additive property, interpretations of chi-square are not limited to $v = 1$. With each mutually exclusive χ^2 or z^2 value computed on several independent measures of X, a chi-square statistic based on a combination of the values can be formed:

$$\chi^2 = \Sigma z^2 = \Sigma(X - M)^2/S^2. \tag{3}$$

Upon reaching $v = 30$ *df*s, the chi-square distribution becomes basically symmetrical and approximates the normal distribution. Obviously, a relationship between chi-square and commonly used parametric tests exists.

In multiple linear regression, the proportion of variance shared between the dependent variable and the predictors is frequently reported as R-square. In the analysis of variance (ANOVA), the proportion of variance of a continuous dependent variable explained by a categorical independent variable is typically estimated and referred to as a treatment magnitude (e.g., η^2, which equals R^2, is a commonly used estimate). Furthermore, when all variables are categorical, a *proportion of shared variance* interpretation is also possible in contingency chi-square applications. That is, the interpretation of the proportion of variance in one variable shared by the other holds for categorical data. Thus, the interpretation of R^2 in the general linear model also carries over to the analysis of contingency tables (Leitner, 1979).

Consider an $R \times C$ contingency table in which $(R - 1)$ and $(C - 1)$ dummy variables, representing row and column membership, respectively, are created. Based on Eckart and Young (1936), Fisher (1940) showed that for any contingency table containing nonzero cells, there must exist H (where $H = \min [R - 1, C - 1]$) pairs of standardized *canonical* variables (x_{ch}, y_{rh}) where $h = 1, 2, \ldots H$. The x values are functions of the columns, and y values are functions of the rows. Across different sets, the x variables are uncorrelated, as are the sets of y. However, for a given set h, there is a canonical correlation between x_{ch} and y_{rh} values, which is simply the Pearson correlation between the x and y values of that set taken over the cell frequencies, f_{rc}, of the contingency table,

$$\rho_h = \sum_{r=1}^{R} \sum_{c=1}^{C} \frac{x_{ch} y_{rh} f_{rc}}{N}. \tag{4}$$

Now consider a contingency table in which the number of columns is either greater than or equal to the number of rows, $R \leq C$, and the $H = (R - 1)$ dummy variables for the rows are transformed into H orthogonal (uncorrelated) variables, d_h, and regressed on the $(C - 1)$ dummy variables representing column membership. The chi-square value from the entire contingency table divided by the total sample size, N, is equal to the sum of the $H = (R - 1)$ values of R^2, each corresponding to the multiple regression of a single row variable d_h on the $(C - 1)$ dummy variables representing columns:

$$\chi^2/N = \sum_{h=1}^{H} R^2_{d_h \cdot \text{columns}} . \tag{5}$$

Therefore, congruent with Equation 4, Equation 5 implies that a chi-square value divided by N is equivalent to the sum of the squared canonical correlations for the H sets of latent variables underlying the contingency table (see Hays, 1988, pp. 793–794) and that statistics calculated from a contingency table analysis can be converted into a proportion of shared variance. Furthermore, because H is the minimum of $(R - 1)$ or $(C - 1)$ and the exact number of canonical correlations, the squared value of the Cramer's statistic, V_C^2 is simply the average squared canonical correlation: $V_C^2 = \chi^2/NH$.

Example From 1990 National Assessment for Educational Progress (NAEP) Policy File

Obtaining a graduate degree has become a common trend among American teachers. Typically, math teachers who were undergraduate education majors continue in a graduate program in education. Teachers who were mathematics majors usually have a choice between obtaining a graduate degree in mathematics or education. As would be expected, the graduate and undergraduate coursework of these teachers with different backgrounds varies considerably.

Another growing issue among teacher education programs is how to prepare prospective teachers for the diversity of the student population. A common approach is to offer and even require preparation courses that involve teaching methods for culturally diverse students and for students with cognitive difficulties. Thus, a question arises as to whether teachers with graduate-level education who have different educational backgrounds are equally prepared for the diversity of the student population.

We obtained data from the 1990 NAEP Mathematics Policy minifile for eighth graders. Math teachers who held graduate degrees were classified into $(R = 3)$ degree categories: undergraduate and graduate degrees in education (EE); undergraduate degree in mathematics, graduate degree in education (ME); and undergraduate and graduate degrees in mathematics (MM). Training classifications were based on the response to two questions: "Were you trained to teach students from different cultures?" and "Were you trained to teach students with different cognitive styles?" Thus, there were $(C = 4)$ training classifications: teachers who were trained in methods for both cultural and cognitive student diversity (BOTH), teachers who were trained to teach students with cognitive difficulties only (COGN), teachers who were trained in cultural diversity only (CULT), and teachers who received neither type of training (NONE). For the purposes of this example, respondents who did not fit into this cross-classification were excluded.

Table 1 contains the cross-tabulated NAEP data. The general null hypothesis is that there is no relationship between these two categorical factors or that math teachers with various educational backgrounds do not differ in the training they have received to prepare for student diversity. The result was a statistically significant omnibus chi-square value ($\chi^2[6, N = 350] = 37.026, p < .00001$), which, divided by N, can be converted to a proportion of shared variance, $R^2 = .106$. Thus, math teachers with various graduate and undergraduate degrees differed in their preparation to deal with student diversity, and their educational background accounted for 10.58% of the variance in training. However, the nature of this relationship and which cell(s) contributed the most to this statistically significant relationship is yet undetermined.

Orthogonal Partitioning of Planned Contrasts

The fact that the chi-square tests can be transformed into measures of association related to the orthogonal latent variables underlying a contingency table (Equation 4) implies that the squared canonical correlations can be transformed into partial chi-square values that sum to the omnibus chi-square value for the entire table (Equation 5). However, the notion of latent variables underlying a contingency table may obscure the meaning of the categorical memberships. But, if a researcher were to create orthogonal contrast codes for each row and column factor based on logical or theoretical grounds, the contingency table could be orthogonally partitioned into v separate single-*df* chi-square tests while preserv-

TABLE 1
Cross-Tabulation of 1990 NAEP Policy Data

	Preparation courses for student diversity				
Educational background	BOTH	COGN	CULT	NONE	Row total
EE	59 (50.143)	35 (45.429)	8 (12.857)	48 (41.571)	$f_{1*} = 150$
ME	23 (31.423)	44 (28.469)	2 (8.057)	25 (26.051)	$f_{2*} = 94$
MM	35 (35.434)	27 (32.103)	20 (9.086)	24 (29.377)	$f_{3*} = 106$
Column total	$f_{*1} = 117$	$f_{*2} = 106$	$f_{*3} = 30$	$f_{*4} = 97$	$N = 350$
$\chi^2(6) = 37.026, p < .00001$					

Note. Expected values are in parentheses. EE = undergraduate and graduate degrees in education; ME = undergraduate degree in mathematics, graduate degree in education; MN = undergraduate and graduate degrees in mathematics.

ing meaningful distinctions between categories. The utility of planned contrasts is widely recognized (Thompson, 1994b).

In the current example, for the educational degree (row) variable, it seemed logical to compare math teachers who have earned graduate degrees in education (i.e., groups EE and ME) with those who received their graduate degrees in mathematics (MM). In ANOVA terminology, this is considered a planned contrast (ψ_i). The null hypothesis is that the *i*th linear combination of population parameters (in this case, proportional makeup, π_k) with known coefficients c_k is equal to zero,

$$H_0\text{: } \psi_i = c_1\pi_1 + c_2\pi_2 + \ldots + c_k\pi_k = 0, \tag{6}$$

where c_k is nonzero for some k and $\Sigma c_k = 0$. The following contrast for row membership is allowable as a way of comparing math teachers with graduate education degrees to those with graduate degrees in mathematics,

$$\psi_{R1} = 1/2\pi_{EE} + 1/2\pi_{ME} - 1\pi_{MM}. \tag{7}$$

Computationally, sample statistics of the two groups with graduate education degrees are combined and compared with the sample statistics of the group with graduate degrees in mathematics.

The comparison, independent of the contrast in Equation 7, is the pairwise contrast of the two groups with graduate degrees in education that were previously combined (EE and ME); those who obtained graduate degrees in mathematics (MM) are excluded and given a zero weight,

$$\psi_{R2} = 1\pi_{EE} - 1\pi_{ME} + 0\pi_{MM}. \tag{8}$$

Although these comparisons are valid and may have more intuitive meaning than the canonical variables that underlie these group memberships, the contrasts in Equation 7 and Equation 8 are not orthogonal because of unequal cell sizes. For a set of contrast vectors to be orthogonal when samples have unequal frequencies (e.g., Pedhazur, 1982, p. 324), a valid linear contrast is restricted to

$$\Sigma f_k c_k = 0. \tag{9}$$

Therefore, to orthogonalize contrasts, one must consider what is being compared, that is, in this instance, *row* marginal totals. For the second row contrast in Equation 8, referred to as R2, those with graduate degrees in mathematics are excluded and given a zero contrast weight. Teachers with graduate degrees in education are compared based on the differences in their undergraduate major. Thus, the simplest way to satisfy Equation 9 is to use $f_{2*} = 94$ as the coefficient for the first group (EE) and $-f_{1*} = -150$ as the coefficient for the second group (ME). That is, row marginal frequencies are *reversed* so that the contrast vector times their respective row marginals sum to zero ($94f_{1*} - 150f_{2*} + 0f_{3*} = 0$). Therefore, the linear contrast in Equation 8 is transformed to yield

$$\psi_{R2} = 94\pi_{EE} - 150\pi_{ME} + 0\pi_{MM}. \quad (10)$$

For the first row contrast, R1 in Equation 7, math teachers with graduate education degrees, regardless of their undergraduate degrees, are combined and have a row marginal frequency of 244 (i.e., $f_{1*} + f_{2*} = 244$, see Table 1). The marginal frequency for those with graduate degrees in mathematics remains $f_{3*} = 106$. Consequently, these marginal frequencies are reversed, so that when these values are multiplied by their respective row marginal frequencies, the vector sums to zero ($106f_{1*} + 106f_{2*} - 244f_{3*} = 0$) and Equation 7 becomes

$$\psi_{R1} = 106\pi_{EE} + 106\pi_{ME} - 244\pi_{MM}. \quad (11)$$

For the column variable (training), it seems logical to compare math teachers who have had no training in either topic related to student diversity (NONE) with the combination of those teachers who have received at least one type of training. Thus, the linear contrast is

$$\psi_{C1} = 1/3\pi_{BOTH} + 1/3\pi_{COGN} + 1/3\pi_{CULT} - 1\pi_{NONE}. \quad (12)$$

To orthogonalize this vector, one should again consider *column* marginal frequencies in light of the comparison being made. The column marginal frequency for the NONE group is $f_{*4} = 97$ (see Table 1). The remaining three categories are pooled and have a combined frequency of 253. These values are reversed so that $97f_{*1} + 97f_{*2} + 97f_{*3} - 253f_{*4} = 0$, and the linear contrast (12) is transformed into

$$\psi_{C1} = 97\pi_{BOTH} + 97\pi_{COGN} + 97\pi_{CULT} - 253\pi_{NONE}. \quad (13)$$

To maintain independence, we gave the group with no training in student diversity a zero weight and excluded it from subsequent analyses. In excluding this group, we considered it logical to compare those with training in BOTH types of student diversity to the combination of those with only one type of training. The linear contrast is

$$\psi_{C2} = 1\pi_{BOTH} - 1/2\pi_{COGN} - 1/2\pi_{CULT} + 0\pi_{NONE}. \quad (14)$$

To orthogonalize Equation 14 in combination with Equation 13, we reversed the frequency of teachers trained in BOTH areas, $f_{*1} = 117$, with the combined frequency of teachers trained in only one of the student diversities, $f_{*2} + f_{*3} = 136$. As a result, similar to that in Equation 10, the contrast is weighted so that $136f_{*1} - 117f_{*2} - 117f_{*3} + 0f_{*4} = 0$, and Equation 14 becomes

$$\psi_{C2} = 136\pi_{BOTH} - 117\pi_{COGN} - 117\pi_{CULT} + 0\pi_{NONE}. \quad (15)$$

Because $C = 4$ categories has $C - 1 = 3dfs$, one comparison independent of Equations 13 and 15 remains, the contrast of math teachers who have only been trained in teaching students with cognitive difficulties (COGN) to those who have only been trained to teach culturally diverse students (CULT):

$$\psi_{C3} = 0\pi_{BOTH} + 1\pi_{COGN} - 1\pi_{CULT} + 0\pi_{NONE}. \qquad (16)$$

To orthogonalize this vector, one must reverse the column marginal frequencies, and the BOTH and NONE groups are excluded and given zero weight. Thus, Equation 16 is transformed such that

$$\psi_{C3} = 0\pi_{BOTH} + 30\pi_{COGN} - 106\pi_{CULT} + 0\pi_{NONE}. \qquad (17)$$

For data analysis purposes, we paired the orthogonalized contrast coefficients with their respective categories for each subject. However, a simpler way to set up these data is shown in the Appendix. Rather than having $N = 350$ data lines in this example, we paired the $R \times C = 12$ cross-classified cells with their respective cell frequencies and contrast codes. We also present SAS and SPSS commands that weight the values by their respective frequencies and perform the analyses.

Analytically, the row variables in Equations 10 and 11, R1 and R2, respectively, are correlated with the three column contrasts C1, C2, and C3 (13, 15, and 17, respectively). Figure 1 contains an edited SPSS printout for this analysis. Because the two row contrasts, as well as the three column contrasts, are internally orthogonal, they have zero correlations. However, the six correlations across row and column contrasts are not necessarily zero and represent the relationships between the row and column contrasts. In fact, because of the orthogonal structure of the row and column contrasts, these six correlations are equated to the $v = 6$ orthogonal single-*df* partitions of the contingency table. Thus, congruent with Equation 5, these six orthogonal correlation coefficients, when squared, summed, and multiplied by N, equal the omnibus chi-square statistic, as reported in Table 2.

The results are further analyzed and displayed in Table 2; interpretations are supplemented by the frequencies and expected values in Table 1. Three findings were statistically significant at the $\alpha = .05$ level; however, we adjusted the nominal alpha (α_{adj}) by the Sidak (1967) method to control for the inflation of the Type I error rate with

$$\alpha_{adj} = 1 - (1 - \alpha)^{1/t}, \qquad (18)$$

where t equals the number of tests. Because $t = 6$ independent tests were performed, $\alpha_{adj} = .0085$. The relationship between the first row contrast (R1) and the third column contrast (C3) was statistically significant and accounted for 5.37% of shared variance (see Table 2 and Figure 1). This result indicates that unlike math teachers with graduate degrees in mathematics, math teachers with graduate degrees in education differed in whether their training was in teaching cognitively different or culturally diverse students. Based on the expected values in Table 1, it appears that teachers with undergraduate and graduate degrees in mathematics were significantly more likely to have received training in cultural

FIGURE 1. Edited SPSS ouput for analysis of Table 1 data.

Variable	Cases	*M*	*SD*
C1	350	.0000	156.9000
C2	350	.0000	107.4000
C3	350	.0000	35.2022
R1	350	.0000	161.1000
R2	350	.0000	99.2869

-- Correlation Coefficients --

	C1	C2	C3	R1	R2
C1	1.0000	.0000	.0000	−.0747	−.0491
C2	.0000	1.0000	.0000	−.0429	−.1665**
C3	.0000	.0000	1.0000	−.2317**	.1207*
R1	−.0747	−.0429	−.2317**	1.0000	.0000
R2	−.0491	−.1665**	.1207**	.0000	1.0000

* - Signif. LE .05 ** - Signif. LE .01 (2-tailed)

EDUC by TRAINING

EDUC / Count / Exp Val / Residual / Std Res	TRAINING BOTH	COGN	CULT	NONE	Row Total
EE	59	35	8	48	150
	50.1	45.4	12.9	41.6	42.9%
	8.9	−10.4	−4.9	6.4	
	1.3	−1.5	−1.4	1.0	
ME	23	44	2	25	94
	31.4	28.5	8.1	26.1	26.9%
	−8.4	15.5	−6.1	−1.1	
	−1.5	2.9	−2.1	−0.2	
MM	35	27	20	24	106
	35.4	32.1	9.1	29.4	30.3%
	−0.4	−5.1	10.9	−5.4	
	−0.1	−0.9	3.6	−1.0	
Column Total	117	106	30	97	350
	33.4%	30.3%	8.6%	27.7%	100.0%

Chi-Square	Value	*df*	Significance
Pearson	37.0262	6	< .00001
Minimum Expected Frequency -	8.057		

diversity and less likely to have received training in cognitive diversity than were those with graduate degrees in education.

The relationship between the second row contrast (R2) and the second column contrast (C2) accounted for 2.77% of shared variance and indicates that among math teachers with graduate education degrees, those with undergraduate degrees in mathematics received training in teaching BOTH cognitively different and cul-

TABLE 2
Results From Orthogonal Partitioning of Data in Table 1

Row contrast	Column contrast	r	R^2	χ^2	p
R1 (11)	C1 (13)	–.0747	.0056	1.932	.1623
	C2 (15)	–.0429	.0018	0.643	.4242
	C3 (17)	–.2317	.0537	18.795	< .0001*
			ΣR^2 = .0611	$\chi^2_{(3)}$ = 21.385	< .0001*
R2	C1 (13)	–.0491	.0024	0.842	.3601
	C2 (15)	–.1665	.0277	9.697	.0018*
	C3 (17)	.1207	.0146	5.098	.0239+
			ΣR^2 = .0447	$\chi^2_{(3)}$ = 15.645	< .0001*
			ΣR^2 = .1058	$\chi^2_{(6)}$ = 37.026	< .0001*

Note. Numbers in parentheses refer row and column contrasts to equations in the text. An asterisk (*) indicates statistical significance at the adjusted alpha level of .0085. The symbol + indicates statistical significance at the nominal alpha level of .05.
[a]N = 350.

turally diverse students significantly less than expected. That is, they were more likely to have received only one type of training, whereas teachers with both graduate and undergraduate degrees in education were more likely to have received BOTH types of training. Also, although the relationship between R2 and C3 did not reach statistical significance at the adjusted alpha level of .0085, its p value was less than the nominal level of .05 and accounted for 1.46% of shared variance. Thus, there is marginal evidence that mathematics majors with graduate degrees in education received training solely in cultural diversity less, and training in cognitive diversity more, than education majors with graduate degrees in education did.

In addition, because the row and column contrasts were orthogonal, the squared correlations and single-*df* chi-square values can be summed to create total contrast effects. For example, by summing the first three squared correlations in Table 2, we find that R1, the contrast of teachers with graduate education degrees to teachers with graduate degrees in mathematics, accounted for 6.11% of the variance in diversity training. This result, which can be converted into a chi-square with $df = 3$ by summing the single-df chi-squares or by multiplying the proportion of shared variance by $N = 350$, was statistically significant, $\chi^2(3, N = 350) = 21.385$, $p < .0001$. Also, among teachers with graduate degrees in education, the R2 contrast between teachers with undergraduate education and mathematics majors was statistically significant, $\chi^2(3, N = 350) = 15.645$, $p = .0013$, and accounted for 4.47% of the variance in diversity training, as noted in Table 2.

Post Hoc Methods

For some contingency tables, either a priori hypotheses do not exist or, after partitioning, the researcher still desires further analysis to understand where differences exist. Therefore, it is possible to augment the omnibus and partitioned chi-squares by post hoc methods.

Standardized Residual Method

The *standardized residual method* is credited to Haberman (1973, 1978) and is later discussed in Marascuilo and McSweeney (1977), Reynolds (1984), and Siegel and Castellan (1988). Because the omnibus chi-square value does not specify which combination of categories contributes to statistical significance, a standardized residual for each cell can be used to determine which discrepancies between observed and expected values are larger than might be expected by chance. It is computed as follows:

$$e_{rc} = \frac{(f_{rc} - E_{rc})}{\sqrt{E_{rc}}} \tag{19}$$

Standardized residuals for each cell are available through the SPSS (1990) CROSSTABS command options. An analogous procedure is to simply take the square root of the cell chi-square values, which are available through options in SAS (1993) PROC FREQ, and keep the sign (positive or negative) of the difference between the observed and expected values. When a standardized residual is greater than 2.00 in absolute value (a rule of thumb suggested by Haberman, 1973), the researcher can conclude that the residual contributes to the overall significant chi-square value.

However, the *greater-than-2.00* rule of thumb is an approximation of the two-tailed critical value of z at the $\alpha = .05$ level of significance. Therefore, we suggest that the unit normal table can be used when evaluating standardized residuals after appropriate adjustment for the Type I error rate. In this case, there are $s = 12$ cell values being tested. Substituting s for t and using Equation 18 results in an α_{adj} of .0043, which when converted to the unit normal table gives a two-tailed critical value of $z_{cv} = 2.86$. Note, however, that with 12 cell tests and 6 *df*s, some of these tests are correlated. Therefore, the tests of statistical significance reported are conservative.

The standardized residuals in two cells, s_{22} and s_{33}, significantly contributed to the significant omnibus chi-square statistic (see Table 3). Thus, math teachers with graduate education degrees and undergraduate majors in mathematics (ME) received training solely in teaching cognitively diverse students (COGN) significantly more than expected (cell s_{22}, Table 3); this result is consistent with the previous analysis. The tendency for these teachers to be trained in cultural diversity (CULT) less than expected was of marginal statistical significance (cell s_{23}). Also congruent with the orthogonal partitioning analyses, teachers with undergraduate and graduate degrees in mathematics (MM) were trained solely in cultural diversity (CULT) significantly more than expected (cell s_{33}).

Relative and Absolute Contribution Methods

The analysis of standardized residuals has been criticized because the results are lacking in discrimination and interpretability. That is, no insight into the relative or absolute contribution of the individual cells is provided. However, a post hoc analysis using the relative and absolute contribution of each cell in Table 3 can be easily performed, especially because the overall chi-square is a sum of the individual cell chi-square values. The *relative contribution* method is computed by dividing each cell chi-square by the omnibus chi-square value; this computation gives a percentage contribution for each cell to the overall test statistic that is easy to interpret and use:

$$\%\text{ relative contribution } = \frac{(f_{rc} - E_{rc})^2}{E_{rc}\chi^2} \times 100. \tag{20}$$

Because chi-square values are converted into analogous R^2 values when divided by N, an *absolute contribution* to the proportion of variance shared by the two factors of an $R \times C$ contingency table can be calculated when each cell chi-square is divided by N:

$$\%\text{ absolute contribution } = \frac{(f_{rc} - E_{rc})^2}{E_{rc}N} \times 100. \tag{21}$$

TABLE 3
Calculation of Chi-Square, Standardized Residuals, and Relative and Absolute Contribution of Data in Table 1

Cell	f_{rc}	Cell chi-square $(f_{rc} - E_{rc})^2/E_{rc}$	Standardized residual	Relative (%) contribution	Absolute (%) contribution
s_{11}	59	1.565	1.251	4.225	0.447
s_{12}	35	2.394	−1.547	6.466	0.684
s_{13}	8	1.835	−1.355	4.956	0.524
s_{14}	48	0.994	0.997	2.685	0.284
s_{21}	23	2.258	−1.503	6.098	0.645
s_{22}	44	8.473	2.911*	22.885	2.421
s_{23}	2	4.554	−2.134⁺	12.298	1.301
s_{24}	25	0.042	−0.206	0.115	0.012
s_{31}	35	0.005	−0.073	0.014	0.002
s_{32}	27	0.811	−0.090	2.191	0.232
s_{33}	20	13.111	3.621*	35.411	3.746
s_{34}	24	0.984	−0.992	2.658	0.281
Total		$\chi^2_{(6)} = 37.026$		100.000%	10.579%
			$R^2 = \chi^2/N = 37.026/350 = .1058$		

Note. An asterisk (*) indicates statistical significance at the adjusted alpha level of .0043. The symbol + indicates statistical significance at the nominal alpha level of .05.

Consistent with the statistically significant findings in the analysis of standardized residuals, cells s_{22} and s_{33} made larger relative contributions (22.89% and 35.41%, respectively) to the statistically significant omnibus chi-square test. Likewise, the low percentages in the other cells add additional meaning to interpreting the chi-square results. In terms of absolute contribution, the finding that the ME group was significantly more likely than expected to have been trained solely in methods for teaching students with cognitive difficulties (cell s_{22}) accounted for 2.42% of the variance shared between education background and preparatory training. The statistically significant standardized residual indicating that teachers with undergraduate and graduate mathematics degrees (MM) were trained solely in cultural diversity (CULT) more than expected accounted for 3.75% of shared variance (cell s_{33}).

We should note that researchers must be cautious in their interpretation of these post hoc methods because standardized residuals are not independent. Also, these methods involve simple transformations of these residuals; therefore, this caveat also stands for interpretation of relative and absolute percentage contributions.

Summary

Because statistical significance in most applications is a function of sample size, we should note that the percentages of absolute shared variance reported in this example are rather small; this result qualifies their interpretation. However, the results from both analyses indicate that math teachers with various educational backgrounds do not differ in their lack of preparation for student diversity (i.e., there are no significant correlations with C1). Rather, their differences lie in the type of training they have received. Basically, math teachers with graduate education degrees who came from mathematics undergraduate backgrounds were less likely to have received training in cultural diversity than math teachers who have obtained both undergraduate and graduate degrees in education. Given the growing importance of multiculturalism in the U.S. education system, the development of appropriate bridge courses for those with limited undergraduate backgrounds in education is an issue that needs to be addressed.

The meaning added to finding a statistically significant relationship between two categorical variables, by calculating the relative and absolute contribution of cells and orthogonal partitions, can provide researchers with a valuable tool to better evaluate and interpret contingency table results. The amount of variance shared between variables in a chi-square analysis can therefore be similar to interpretations found in correlation and regression. Although the example was based on evaluating the relationship of two variables, the technique can also be used for one-dimensional chi-square analyses (i.e., goodness-of-fit). Also, given that contingency tables can be orthogonally partitioned, variables measured on an ordinal or interval scale may be analyzed by using orthogonal polynomial trend coefficients. In summary, these methodologies present flexible procedures for analyzing contingency table data that supplement techniques already available. The extension of these methods to ordinal data and multidimensional tables warrants further consideration.

NOTE

Send correspondence to T. Mark Beasley, School of Education, St. John's University, 8000 Utopia Parkway, Jamaica, NY 11439. E-mail beasleyt@sjuvm.stjohns.edu

REFERENCES

Best, J. W. (1981). *Research in education* (4th ed.). Englewood Cliffs, NJ: Prentice-Hall.

Burke, C. J. (1951). Letter to the editor of Peter's reply to Lewis and Burke. *Psychological Bulletin, 48*, 81–82.

Eckart, C., & Young, G. (1936). The approximation of one matrix by another of lower rank. *Psychometrika, 1*, 211–218.

Fisher, R. A. (1940). The precision of discriminant functions. *Annals of Eugenics, 10*, 422–429.

Glass, G. V, & Stanley, J. C. (1970). *Statistical methods in education and psychology* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Guilford, J. P., & Fruchter, B. (1973). *Fundamental statistics in psychology and education* (4th ed.). New York: McGraw-Hill.

Haberman, S. J. (1973). The analysis of residuals in cross-classified tables. *Biometrics, 29*, 205–220.

Haberman, S. J. (1978). *Analysis of qualitative data* (Vol. 1). London: Academic Press.

Hays, W. L. (1988). *Statistics* (4th ed.). New York: Holt, Rinehart, & Winston.

Kerlinger, F. N. (1973). *Foundations of behavioral research* (2nd ed.). New York: Holt, Rinehart, & Winston.

Leitner, D. W. (1979). Using multiple regression to interpret chi-square contingency table analysis. *Multiple Linear Regression Viewpoints, 10*, 39–45.

Lewis, D., & Burke, C. L. (1949). The use and misuse of the chi-square test. *Psychological Bulletin, 46*, 433–489.

Marascuilo, L. A. (1966). Large sample multiple comparisons. *Psychological Bulletin, 65*, 280–290.

Marascuilo, L. A., & McSweeney, M. (1977). *Nonparametric and distribution free methods for the social sciences*. Monterey, CA: Brooks/Cole.

Pedhazur, E. J. (1982). *Multiple regression in behavioral research: Explanation and prediction* (2nd ed.). New York: Holt, Rinehart, & Winston.

Reynolds, H. T. (1984). *Analysis of nominal data*. Newbury Park, CA: Sage.

SAS Institute Inc. (1993). *SAS* (*Version 7*): *Users' guide*. Cary, NC: Author.

Sidak, Z. (1967). Rectangular confidence regions for the means of multivariate normal distributions. *Journal of the American Statistical Association, 62*, 626–633.

Siegel, S., & Castellan, N. J. (1988). *Nonparametric statistics for the behavioral sciences* (2nd ed.). New York: McGraw-Hill.

SPSS Inc. (1990). *SPSS reference guide*. Chicago: Author.

Thompson, B. (1988). Misuse of chi-square contingency-table test statistics. *Educational and Psychological Research, 8*, 39–49.

Thompson, B. (1994a). Guidelines for authors. *Educational and Psychological Measurement, 54*(4), 837–847.

Thompson, B. (1994b). Planned versus unplanned and orthogonal versus nonorthogonal contrasts: The neo-classical perspective. In B. Thompson (Ed.), *Advances in social science methodology* (Vol. 3, pp. 3–27). Greenwich, CT: JAI Press.

APPENDIX
SAS and SPSS Setup Commands

SAS Commands and Data for Table 1

```
data one; options ls = 73;
input cell $ TRAINING $ EDUC $ frc CI C2 C3 RI R2;
cards;
s11 BOTH   EE   59   +97   +136     0   +106   +94
s12 COGN   EE   35   +97   -117   +30   +106   +94
s13 CULT   EE    8   +97   -117  -106   +106   +94
s14 NONE   EE   48  -253      0     0   +106   +94
s21 BOTH   ME   23   +97   +136     0   +106  -150
s22 COGN   ME   44   +97   -117   +30   +106  -150
s23 CULT   ME    2  -253   -117  -106   +106  -150
s24 NONE   ME   25  -253      0     0   +106  -150
s31 BOTH   MM   35   +97   +136     0   -244     0
s32 COGN   MM   27   +97   -117   +30   -244     0
s33 CULT   MM   20   +97   -117  -106   -244     0
s34 NONE   MM   24  -253      0     0   -244     0
proc freq; tables EDUC*TRAINING/chisq cellchi2 expected deviation;weight frc;
proc corr; var C1 C2 C3 R1 R2;freq frc;
```

SPSS Command for Analysis of Data in Table 1

```
data list/cell 1-3 (a) training 5-8 (a) educ 11-12 (a) frc 14-16 C1 18-20 C2 23-25
   C3 28-30 R1 33-35 R2 38-41.
begin data
.
.
.
end data.
weight by frc.
correlations variables = C1 C2 C3 R1 R2
   /statistics = descriptives.
crosstabs tables = educ by training
   /cells = all
   /statistics = chisq.
```