Simple Regression through General Linear Hypothesis Testing

	Y		X	
	1		1	2
	3		1	4
	2		1	3
	5		1	7
	4		1	6
	7		1	8
	6		1	5
Sum	28		7	35
Mea	4		1	5
USS	140		7	203
CSS	28		0	28

$$\mathbf{X'X} = \begin{bmatrix} 7 & 35 \\ 35 & 203 \end{bmatrix}$$

$$(\mathbf{X`X})^{-1} = \begin{bmatrix} 1.0357 & -0.1786 \\ -0.1786 & 0.0357 \end{bmatrix}$$

0.0357 ← $1/28$ = CSS_{X1}^{-1}

$$\mathbf{X'y} = 28 \quad 165$$

$$\hat{\beta} = (\mathbf{X`X})^{-1}\mathbf{X'y} = \begin{matrix} -0.46429 = \hat{\beta}_0 \\ 0.89286 = \hat{\beta}_1 \end{matrix}$$

Define $\mathbf{C} = [\ 0\ 1\ ]$

$$\hat{\theta} = \mathbf{C}\hat{\beta} = \hat{\beta}_1 \qquad \mathbf{M} = \mathbf{C}(\mathbf{X`X})^{-1}\mathbf{C'} = CSS_X^{-1} = 1/28$$

$$\mathbf{M}^{-1} = CSS_X = 28$$

$$SS_H = \hat{\theta}\mathbf{M}^{-1}\hat{\theta}' = (\mathbf{C}\hat{\beta})\mathbf{M}^{-1}(\mathbf{C}\hat{\beta})' = \hat{\beta}_1 CSS_{X1}\hat{\beta}_1' = \hat{\beta}_1^2 CSS_{X1}$$

$$F = \frac{SS_H/a}{MS_E}$$

With a =1

$$F = \hat{\beta}_1^2 CSS_{X1}/MS_E$$

$$SS_H = \sum_{i=1}^{N}(\hat{y}_i - \bar{y})^2 \quad \text{where} \quad \hat{y}_i = \mathbf{X}\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{X}_1$$

$$\hat{y}_i = \bar{y} - \hat{\beta}_1 \bar{X}_1 + \hat{\beta}_1 X_{1i}$$

$$\hat{y}_i = \bar{y} + \hat{\beta}_1 (X_{1i} - \bar{X}_1)$$

$$(\hat{y}_i - \bar{y}) = \bar{y} + \hat{\beta}_1 (X_{1i} - \bar{X}_1) - \bar{y}$$

$$(\hat{y}_i - \bar{y}) = \hat{\beta}_1 (X_{1i} - \bar{X}_1)$$

$$SS_H = \sum_{i=1}^{N}(\hat{y}_i - \bar{y})^2 = \sum_{i=1}^{N} [\hat{\beta}_1 (X_{1i} - \bar{X}_1)]^2 = \hat{\beta}_1^2 \sum_{i=1}^{N} (X_{1i} - \bar{X}_1)^2 = \hat{\beta}_1^2 CSS_X$$

$$F = t^2 = \hat{\beta}_1^2 / SE^2_{(\hat{\beta}_1)} \qquad SE^2_{(\hat{\beta}_1)} = MS_E/[CSS_{X1}(Tolerance_j)]$$

$$F = t^2 = \hat{\beta}_1^2 [CSS_{X1}(Tolerance_j)]/MS_E$$; Tolerance = 1 with one predictor

$$F = t^2 = \hat{\beta}_1^2 [CSS_X]/MS_E$$

Multiple Regression Test of a Single Variable through General Linear Hypothesis Testing

	Y		X		
	1		1	2	6
	3		1	4	8
	2		1	3	12
	5		1	7	10
	4		1	6	14
	7		1	8	18
	6		1	5	16
Sum	28		7	35	84
Mean	4		1	5	12
USS	140		7	203	1120
CSS	28		0	28	112

Z′Z = CSSCP	y	x_1	x_2
	28	25	46
	25	28	38
	46	38	112

X′X =

7	35	84
35	203	458
84	458	1120

$(X^{\prime}X)^{-1}$ =

1.4856	-0.06147	-0.0863
-0.0615	**0.06619**	-0.0225
-0.0863	-0.02246	0.0165

(0.06619 → 1/15.1071 = RSS_{X1}^{-1})

X′y = 28 165 382

$\hat{\beta} = (X^{\prime}X)^{-1}X^{\prime}y =$
-1.5059 = $\hat{\beta}_0$
0.6218 = $\hat{\beta}_1$
0.1998 = $\hat{\beta}_2$

Define **C** = [0 1 0]

$\hat{\theta} = \mathbf{C}\hat{\beta} = \hat{\beta}_1$

$M = \mathbf{C}(X^{\prime}X)^{-1}\mathbf{C}^{\prime} = RSS_X^{-1} = 1/15.1071$

$M^{-1} = RSS_X = 15.1071$

$SS_H = \hat{\theta}M^{-1}\hat{\theta}^{\prime} = (\mathbf{C}\hat{\beta})M^{-1}(\mathbf{C}\hat{\beta})^{\prime} = \hat{\beta}_1 RSS_{X1}\hat{\beta}_1^{\prime} = \hat{\beta}_1^2 RSS_{X1}$

$F = \dfrac{SS_H/a}{MS_E}$

With a =1
$MS_E = 0.8168$

$F = \hat{\beta}_1^2 RSS_{X1}/MS_E$

$\dfrac{0.6218^2(15.1071)}{0.8168} = 7.151$

Tolerance = $1 - R^2_{12} = 1 - [38/\{(28)^{½}(112)^{½}\}]^2$
$= 1 - (.6786)^2$
$= .5395$

$F = t^2 = \hat{\beta}_1^2/SE^2_{(\hat{\beta}_1)}$ $\quad SE^2_{(\hat{\beta}_1)} = MS_E/[CSS_{X1}(Tolerance_j)]$

$SE^2_{(\hat{\beta}_1)} = 0.8168/[(28)(0.5395)] = 0.8168/15.1071 = 0.05406$

$F = t^2 = \hat{\beta}_1^2/SE^2_{(\hat{\beta}_1)} = 0.6218^2/0.05406 = 7.151$

$F = t^2 = \hat{\beta}_1^2[CSS_{X1}(Tolerance_j)]/MS_E$

$F = t^2 = [0.6218^2(28)(0.5395)]/0.8168 = 7.151$