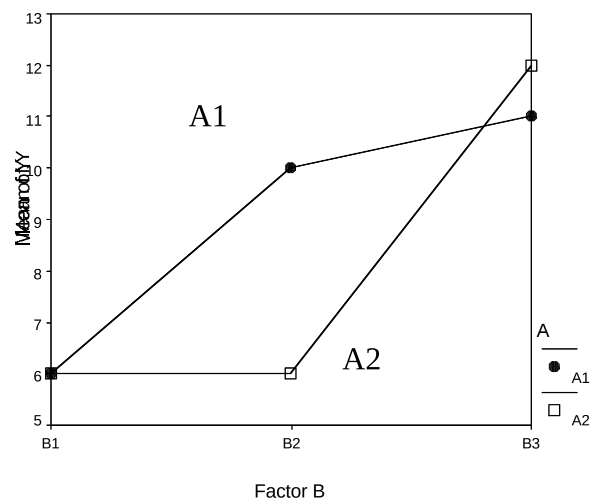
Two-Way ANOVA

In the Two-Way (Factorial) ANOVA, the two factors represent separate sources of variance. The interaction of these two factors also presents an independent source of variation. Suppose a design in which a Factor A has two levels (e.g., Treatment vs. Control; Male vs. Female) and Factor B has three levels (e.g., High, Medium, and Low ability groups; Control, Treatment 1, and Treatment 2). The design has 2x3=6 cell (group) means.

	B_1		B_2		B_3		Total
A_1		$(Y-\bar{Y}_{11})^2$		$(Y-\bar{Y}_{12})^2$		$(Y-\bar{Y}_{13})^2$	
	9	$(9 - 6)^2$	12	$(12-10)^2$	13	$(13-11)^2$	
	7	$(7 - 6)^2$	12	$(12-10)^2$	12	$(12-11)^2$	
	5	$(5 - 6)^2$	11	$(11-10)^2$	10	$(10-11)^2$	
	3	$(3 - 6)^2$	5	$(5-10)^2$	9	$(9-11)^2$	$n_{1*}=12$
	$\bar{Y}_{11}$ = **6**	$SS_{W(11)}=20$	$\bar{Y}_{12}$ = **10**	$SS_{W(12)}=34$	$\bar{Y}_{13}$ = **11**	$SS_{W(13)}=10$	$\bar{Y}_{1*}$ = **9**
A_2		$(Y-\bar{Y}_{21})^2$		$(Y-\bar{Y}_{22})^2$		$(Y-\bar{Y}_{23})^2$	
	7	$(7 - 6)^2$	8	$(8 - 6)^2$	14	$(14-12)^2$	
	7	$(7 - 6)^2$	7	$(7 - 6)^2$	13	$(13-12)^2$	
	5	$(5 - 6)^2$	5	$(5 - 6)^2$	11	$(11-12)^2$	
	5	$(5 - 6)^2$	4	$(4 - 6)^2$	10	$(10-12)^2$	$n_{2*}=12$
	$\bar{Y}_{21}$ = **6**	$SS_{W(21)}=4$	$\bar{Y}_{22}$ = **6**	$SS_{W(22)}=10$	$\bar{Y}_{23}$ = **12**	$SS_{W(23)}=10$	$\bar{Y}_{2*}$ = **8**
Total	$\bar{Y}_{*1}$ = **6** $n_{*1} = 8$		$\bar{Y}_{*2}$ = **8** $n_{*2} = 8$		$\bar{Y}_{*3}$ = **11.5** $n_{*3} = 8$		$\bar{Y}_{**}$ =**8.5** $N = 24$

TWO-WAY ANOVA MODEL: $Y_{ijk} = \mu^{**} + \alpha_j + \beta_k + \alpha\beta_{jk} + \varepsilon_{ijk}$



Factor A has two marginal means, $\bar{Y}_{1*}$ and $\bar{Y}_{2*}$, and (A-1=2-1=1) degree of freedom.

The null hypothesis for Factor A is $H_0: \mu_{1*} = \mu_{2*}$

or $H_0: \Sigma\alpha_j^2 = 0$.

Factor B has three marginal means, $\bar{Y}_{*1}$, $\bar{Y}_{*2}$, and $\bar{Y}_{*3}$, and (B-1=3-1=2) degrees of freedom.

The null hypothesis for Factor B is $H_0: \mu_{*1} = \mu_{*2} = \mu_{*3}$

or $H_0: \Sigma\beta_k^2 = 0$.

The interaction term is multiplicative conceptually; thus, the AxB interaction has (A-1)x(B-1) = [(2-1)x(3-1)] = 2 degrees of freedom.

The null hypothesis is quite complex, $H_0: (\mu_{11} - \mu_{21}) = (\mu_{12} - \mu_{22}) = (\mu_{13} - \mu_{23})$

or $H_0: \Sigma\alpha\beta^2_{jk} = 0$.

It implies that the absence of an interaction indicates that the main effects of Factor A are independent of the main effects of Factor B. Since interactions are symmetric, the absence of an interaction also indicates that the main effects of Factor B are independent of the main effects of Factor A.

SPSS Results

	Y	Group	A	B	X1	X2	X3	X4	X5
1	9	1	A1	B1	1	2	0	2	0
2	7	1	A1	B1	1	2	0	2	0
3	5	1	A1	B1	1	2	0	2	0
4	3	1	A1	B1	1	2	0	2	0
5	12	2	A1	B2	1	-1	1	-1	1
6	12	2	A1	B2	1	-1	1	-1	1
7	11	2	A1	B2	1	-1	1	-1	1
8	5	2	A1	B2	1	-1	1	-1	1
9	13	3	A1	B3	1	-1	-1	-1	-1
10	12	3	A1	B3	1	-1	-1	-1	-1
11	10	3	A1	B3	1	-1	-1	-1	-1
12	9	3	A1	B3	1	-1	-1	-1	-1
13	7	4	A2	B1	-1	2	0	-2	0
14	7	4	A2	B1	-1	2	0	-2	0
15	5	4	A2	B1	-1	2	0	-2	0
16	5	4	A2	B1	-1	2	0	-2	0
17	8	5	A2	B2	-1	-1	1	1	-1
18	7	5	A2	B2	-1	-1	1	1	-1
19	5	5	A2	B2	-1	-1	1	1	-1
20	4	5	A2	B2	-1	-1	1	1	-1
21	14	6	A2	B3	-1	-1	-1	1	1
22	13	6	A2	B3	-1	-1	-1	1	1
23	11	6	A2	B3	-1	-1	-1	1	1
24	10	6	A2	B3	-1	-1	-1	1	1

Descriptive Statistics from a One-Way ANOVA

GROUP	Mean	N	Std. Dev.
1	6.00	4	2.58
2	10.00	4	3.37
3	11.00	4	1.83
4	6.00	4	1.15
5	6.00	4	1.83
6	12.00	4	1.83
Total	8.50	24	3.27

Descriptive Statistics from a Two-Way ANOVA

A	B	Mean	Std. Dev.	N
A1	B1	6.00	2.58	4
	B2	11.00	3.37	4
	B3	11.00	1.83	4
	Total	9.00	3.30	12
A2	B1	6.00	1.15	4
	B2	6.00	1.83	4
	B3	12.00	1.83	4
	Total	8.00	3.30	12
Total	B1	6.00	1.85	8
	B2	8.00	3.30	8
	B3	11.50	1.77	8
	Total	8.50	3.27	24

```
Output from SAS PROC GLM;
proc glm;class a b;model y = a b a*b;lsmeans a*b /slice=b;run;

Dependent Variable: y
                                  Sum of
 Source               DF         Squares      Mean Square    F Value    Pr > F
 Model                 5     158.0000000      31.6000000       6.46     0.0013
 Error                18      88.0000000       4.8888889
 Corrected Total      23     246.0000000

                    R-Square     Coeff Var      Root MSE        y Mean
                    0.642276      26.01274      2.211083      8.500000

 Source          DF       Type III SS      Mean Square    F Value    Pr > F      Eta^2
 a                1         6.0000000        6.0000000       1.23    0.2825    0.0244
 b                2       124.0000000       62.0000000      12.68    0.0004    0.5041
 a*b              2        28.0000000       14.0000000       2.86    0.0832    0.1138

                      a*b Effect Sliced by b for y
                          Sum of
 b            DF         Squares     Mean Square    F Value     Pr > F
 B1            1    8.874685E-31    8.874685E-31       0.00     1.0000
 B2            1       32.000000       32.000000       6.55     0.0197
 B3            1        2.000000        2.000000       0.41     0.5305
```

Computation of the Two-Way ANOVA Source Table with Between-Subjects Factors

The computation of the Sums of Squares (SS) for the main effects of Factors A and B are similar to the one-way analysis. Marginal means are subtracted from the grand mean, squared, weighted by the marginal sample size, and summed. The interaction SS involve subtracting the cell means from the grand mean and also subtracting the main effects of A and B, squaring, weighting by cell sample size, and summing over all cells. There is an algebraic simplification of this formula.

One-Way ANOVA Linear Parameter Model: $Y_{ij} = \mu^{**} + \gamma_j + \varepsilon_{ij}$

Source		Sum of Squares	*df*	Mean Square	*F*	$\eta 2$
Between- Groups One-Way Model	$(\Sigma\gamma^2_j)$ $g_1 = -2.5$ $g_2 = 1.5$ $g_3 = 2.5$ $g_4 = -2.5$ $g_5 = -2.5$ $g_6 = 3.5$	$\Sigma n_{AB}(\bar{Y}_{AB} - \bar{Y}_{**})^2$ $4(6 - 8.5)^2$ $+ 4(10 - 8.5)^2$ $+ 4(11 - 8.5)^2$ $+ 4(6 - 8.5)^2$ $+ 4(6 - 8.5)^2$ $+ 4(12 - 8.5)^2 =$ **158**	$(AB - 1)$ $(6 - 1) =$ **5**	$SS_G/(AB- 1)$	MS_G/MS_W 158/4.89 = **6.464**	SS_G/SS_T 158/246 = **.642**
Two-Way ANOVA Linear Parameter Model: $Y_{ijk} = \mu^{**} + \alpha_j + \beta_k + \alpha\beta_{jk} + \varepsilon_{ijk}$						
Factor A	$(\Sigma\alpha^2_j)$ $a_1 = 0.5$ $a_2 = -0.5$	$\Sigma n_A(\bar{Y}_{A*} - \bar{Y}_{**})^2$ $12(9-8.5)^2$ $+ 12(8-8.5)^2 =$ **6**	$(A - 1)$ $(2 - 1) =$ **1**	$SS_A/(A- 1)$ 6/1 = **6**	MS_A/MS_W 6/4.889 = **1.227**	SS_A/SS_T 6/246 = **.024**
Factor B	$(\Sigma\beta^2_k)$ $b_1 = -2.5$ $b_2 = -0.5$ $b_3 = 3 .0$	$\Sigma n_B(\bar{Y}_{*B} - \bar{Y}_{**})^2$ $8(6-8.5)^2$ $+ 8(8-8.5)^2$ $+ 8(11.5-8.5)^2 =$ **124**	$(B - 1)$ $(3 - 1) =$ **2**	$SS_B/(B- 1)$ 124/2=**62**	MS_B/MS_W 62/4.889 = **12.682**	SS_B/SS_T 124/246 = **.504**
Interaction (AxB)	$(\Sigma\alpha\beta^2_{jk})$ $ab_{11} = -0.5$ $ab_{12} = 1.5$ $ab_{13} = -1.0$ $ab_{21} = 0.5$ $ab_{22} = -1.5$ $ab_{23} = 1.0$	$\Sigma n_{AB}(\bar{Y}_{AB} - \bar{Y}_{A*} - \bar{Y}_{*B} + \bar{Y}_{**})^2$ $4(6 - 9 - 6 + 8.5)^2$ $+ 4(10 - 9 - 8 + 8.5)^2$ $+ 4(11 - 9 - 11.5 + 8.5)^2$ $+ 4(6 - 8 - 6 + 8.5)^2$ $+ 4(6 - 8 - 8 + 8.5)^2$ $+ 4(12 - 8 - 11.5 + 8.5)^2 =$ **28**	$(A-1)(B-1)$ (2-1)(3-1) (1)(2) = **2**	SS_{AB}/df_{AB} 28/2 = **14**	MS_{AB}/MS_W 14/4.889 = **2.864**	SS_G/SS_T 28/246 = **.114**
Within Groups (Error)	$(\Sigma\varepsilon^2_{ijk})$	$\Sigma(Y_i - \bar{Y}_{AB})^2$ 20+34+10+4+10+10 = **88**	$N - AB$ 24-6 = **18**	SS_W/df_W (88/18 = **4.889**)		
Total Variance		$\Sigma(Y_i - \bar{Y}_{**})^2$ = **246**	$N - 1$ 24 -1 = **23**	($s^2 = S_T/N$-1 = **10.70**)		

where, N = total number of cases, A= number of groups for Factor A, B= number of groups for Factor B, $\bar{Y}_{**}$ = the grand mean of Y across all groups. Y_i = each individual score on Y, n_A = the number of cases in each group of Factor A, n_B = the number of cases in each group of Factor B, and n_{AB} = the number of cases in AB cell.

Treatment magnitude for each effect can calculated for each effect. For example, the eta-squared for Factor B is $R^2 = \eta^2 = SS_B/SS_T = .504$.

General Linear Model Approach

Y_{ijk} = μ_*	$+ \alpha_j$	$+ \beta_k$	$+ \alpha\beta_{jk}$	$+ \varepsilon_{ij}$
Y_{ijk} = $\bar{Y}_*$	$+ a_j$	$+ b_k$	$+ ab_{jk}$	$+ e_{ij}$
9 = 8.5	+0.5	-2.5	-0.5	+ 3
7 = 8.5	+0.5	-2.5	-0.5	+ 1
5 = 8.5	+0.5	-2.5	-0.5	- 1
3 = 8.5	+0.5	-2.5	-0.5	- 3
12 = 8.5	+0.5	-0.5	+1.5	+ 2
12 = 8.5	+0.5	-0.5	+1.5	+ 2
11 = 8.5	+0.5	-0.5	+1.5	+ 1
5 = 8.5	+0.5	-0.5	+1.5	- 5
13 = 8.5	+0.5	+3.0	-1.0	+ 2
12 = 8.5	+0.5	+3.0	-1.0	+ 1
10 = 8.5	+0.5	+3.0	-1.0	- 1
9 = 8.5	+0.5	+3.0	-1.0	- 2
7 = 8.5	+0.5	-2.5	+0.5	+ 1
7 = 8.5	-0.5	-2.5	+0.5	+ 1
5 = 8.5	-0.5	-2.5	+0.5	- 1
5 = 8.5	-0.5	-2.5	+0.5	- 1
8 = 8.5	-0.5	-0.5	-1.5	+ 2
7 = 8.5	-0.5	-0.5	-1.5	+ 1
5 = 8.5	-0.5	-0.5	-1.5	- 1
4 = 8.5	-0.5	-0.5	-1.5	- 2
14 = 8.5	-0.5	+3.0	+1.0	+ 2
13 = 8.5	-0.5	+3.0	+1.0	+ 1
11 = 8.5	-0.5	+3.0	+1.0	- 1
10 = 8.5	-0.5	+3.0	+1.0	- 2
SS_Y =	$\Sigma\alpha^2$ +	$\Sigma\beta^2$ +	$\Sigma\alpha\beta^2$ +	$\Sigma\varepsilon^2$
$\Sigma (Y_{ij} - \mu_{**})^2$	$= \Sigma (\mu_{j*} - \mu_{**})^2$	$= \Sigma (\mu_{*k} - \mu_{**})^2$	$= \Sigma(\mu_{jk} - \mu_{j*} - \mu_{*k} + \mu_{**})^2$	$+ \Sigma (Y_{ij} - \mu_{jk})^2$
$\Sigma (Y_{ij} - \bar{Y}_{**})^2$	$= \Sigma (\bar{Y}_{j*} - \bar{Y}_{**})^2$	$= \Sigma (\bar{Y}_{*k} - \bar{Y}_{**})^2$	$= \Sigma(\bar{Y}_{jk} - \bar{Y}_{j*} - \bar{Y}_{*k} + \bar{Y}_{**})^2$	$+ \Sigma (Y_{ij} - \bar{Y}_{jk})^2$
$(9 - 8.5)^2$	$(9 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 9 - 6 + 8.5)^2$	$(9 - 6)^2$
$(7 - 8.5)^2$	$(9 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 9 - 6 + 8.5)^2$	$(7 - 6)^2$
$(5 - 8.5)^2$	$(9 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 9 - 6 + 8.5)^2$	$(5 - 6)^2$
$(3 - 8.5)^2$	$(9 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 9 - 6 + 8.5)^2$	$(3 - 6)^2$
$(12 - 8.5)^2$	$(9 - 8.5)^2$	$(8 - 8.5)^2$	$(10 - 9 - 8 + 8.5)^2$	$(12 - 10)^2$
$(12 - 8.5)^2$	$(9 - 8.5)^2$	$(8 - 8.5)^2$	$(10 - 9 - 8 + 8.5)^2$	$(12 - 10)^2$
$(11 - 8.5)^2$	$(9 - 8.5)^2$	$(8 - 8.5)^2$	$(10 - 9 - 8 + 8.5)^2$	$(11 - 10)^2$
$(5 - 8.5)^2$	$(9 - 8.5)^2$	$(8 - 8.5)^2$	$(10 - 9 - 8 + 8.5)^2$	$(5 - 10)^2$
$(13 - 8.5)^2$	$(9 - 8.5)^2$	$(11.5 - 8.5)^2$	$(11 - 9 - 11.5 + 8.5)^2$	$(13 - 11)^2$
$(12 - 8.5)^2$	$(9 - 8.5)^2$	$(11.5 - 8.5)^2$	$(11 - 9 - 11.5 + 8.5)^2$	$(12 - 11)^2$
$(10 - 8.5)^2$	$(9 - 8.5)^2$	$(11.5 - 8.5)^2$	$(11 - 9 - 11.5 + 8.5)^2$	$(10 - 11)^2$
$(9 - 8.5)^2$	$(9 - 8.5)^2$	$(11.5 - 8.5)^2$	$(11 - 9 - 11.5 + 8.5)^2$	$(9 - 11)^2$
$(7 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 8 - 6 + 8.5)^2$	$(7 - 6)^2$
$(7 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 8 - 6 + 8.5)^2$	$(7 - 6)^2$
$(5 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 8 - 6 + 8.5)^2$	$(5 - 6)^2$
$(5 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8.5)^2$	$(6 - 8 - 6 + 8.5)^2$	$(5 - 6)^2$
$(8 - 8.5)^2$	$(8 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8 - 8 + 8.5)^2$	$(8 - 6)^2$
$(7 - 8.5)^2$	$(8 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8 - 8 + 8.5)^2$	$(7 - 6)^2$
$(5 - 8.5)^2$	$(8 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8 - 8 + 8.5)^2$	$(5 - 6)^2$
$(4 - 8.5)^2$	$(8 - 8.5)^2$	$(8 - 8.5)^2$	$(6 - 8 - 8 + 8.5)^2$	$(4 - 6)^2$
$(14 - 8.5)^2$	$(8 - 8.5)^2$	$(11.5 - 8.5)^2$	$(12 - 8 - 11.5 + 8.5)^2$	$(14 - 12)^2$
$(13 - 8.5)^2$	$(8 - 8.5)^2$	$(11.5 - 8.5)^2$	$(12 - 8 - 11.5 + 8.5)^2$	$(13 - 12)^2$
$(11 - 8.5)^2$	$(8 - 8.5)^2$	$(11.5 - 8.5)^2$	$(12 - 8 - 11.5 + 8.5)^2$	$(11 - 12)^2$
$(10 - 8.5)^2$	$(8 - 8.5)^2$	$(11.5 - 8.5)^2$	$(12 - 8 - 11.5 + 8.5)^2$	$(10 - 12)^2$
(SS_Y **=246**)	= (SS_A = **6**)	= (SS_B = **124**)	= (SS_{AB} = **= 28**)	= (SS_E = **= 88**)

Post-Hoc Analysis in Factorial Designs

In the absence of a significant AxB Interaction the Post Hoc methods used in the one-way ANOVA can be applied to the marginal means of a significant main effect. Based on the previous example, if the AxB Interaction does not reach statistical significance, but the main effect of Factor A is statistically significant, Tukey's HSD procedure can be applied to the marginal means of A, $\overline{Y}_{1*}$ and $\overline{Y}_{2*}$.

If the AxB Interaction effect is statistically significant, the most common method for Post Hoc analysis is called **Simple Main Effects**. This involves testing for significant differences of one Factor at each level of the other factor. From the previous example, we could test the Simple Main Effect of B at A_1. This would involve the comparison of $\overline{Y}_{11}$, $\overline{Y}_{12}$, and $\overline{Y}_{13}$. The Sums of Squares for B at A_1 is calculated using the marginal mean for A_1, $\overline{Y}_{1*}$. (SS $A_{B\text{ at }A1}$) = $\Sigma n_{A1}(\overline{Y}_{1B} - \overline{Y}_{1*})^2$. The Error Term ($MS_W$) from the original source table would be used to form an F-ratio with B-1=2 and df_W degrees of freedom. . This would also be computed for the other level of A. In this case, if a statistically significant difference is found post-hoc methods are still needed because there are more than two groups.

Therefore, it may be more efficient to test Simple Main Effects of A at each level of B. That is, testing A at B_1 indicates whether $\overline{Y}_{11}$ is significantly different from $\overline{Y}_{21}$. Testing A at B_2 indicates whether $\overline{Y}_{12}$ is significantly different from $\overline{Y}_{22}$. Testing A at B_3 indicates whether $\overline{Y}_{13}$ is significantly different from $\overline{Y}_{23}$.

Source	SS	*df*	MS	*F*	η^2
Main Effects					
Factor A	6	1	6	1.227	.0244
Factor B	124	2	62	12.682*	.5041
Interaction AxB	28	2	14	2.864*	.1138
Simple Main Effects					
A at B_1	0	1	0	0.000	
A at B_2	32	1	32	6.545*	
A at B_3	2	1	2	0.409	
Within Groups (Error Variance)	88	18	4.89		
Total Variance	246	23	10.70		

* indicates significant at the α = .10 level.

In the example presented, it appears that A_1 and A_2 are equivalent at levels B_1 and B_3, therefore, the significant interaction occurs between A_1 and A_2 at B_2., $F(1,18) = 6.545$.

Regression Approach via One-Way ANOVA with Contrasts
Group: 1 = A1-B1; 2 = A1-B2; 3 = A1-B3;
4 = A2-B1; 5 = A2-B2; 6 = A2-B3.

```
proc glm;class group;model y = group;
contrast 'Main effect A' group 1 1 1 -1 -1 -1;
contrast 'Main Effect B'
            group 2 -1 -1  2 -1 -1 ,
            group 0  1 -1  0  1 -1 ;
contrast 'A*B Interaction'
            group 2 -1 -1 -2  1  1 ,
            group 0  1 -1  0 -1  1 ;
contrast 'Simple Effect A @ B1'
            group 1  0  0 -1  0  0;
contrast 'Simple Effect A @ B2'
            group 0  1  0  0 -1  0;
contrast 'Simple Effect A @ B2'
            group 0  0  1  0  0 -1;
contrast 'Simple Effect B @ A1'
            group 2 -1 -1  0  0  0 ,
            group 0  1 -1  0  0  0 ;
contrast 'Simple Effect B @ A2'
            group 0  0  0  2 -1 -1 ,
            group 0  0  0  0  1 -1  ;run;
```

```
     Class          Levels    Values
     group               6    1 2 3 4 5 6
    Number of Observations Used          24
```

Dependent Variable: y — The GLM Procedure

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	**5**	**158.0000000**	**31.6000000**	**6.46**	**0.0013**
Error	**18**	**88.0000000**	**4.8888889**		
Corrected Total	**23**	**246.0000000**			

R-Square	Coeff Var	Root MSE	y Mean
0.642276	**26.01274**	**2.211083**	**8.500000**

Source	DF	Type III SS	Mean Square	F Value	Pr > F
group	**5**	**158.0000000**	**31.6000000**	**6.46**	**0.0013**

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Main effect A	**1**	**6.0000000**	**6.0000000**	**1.23**	**0.2825**
Main Effect B	**2**	**124.0000000**	**62.0000000**	**12.68**	**0.0004**
A*B Interaction	**2**	**28.0000000**	**14.0000000**	**2.86**	**0.0832**
Simple Effect A @ B1	**1**	**0.0000000**	**0.0000000**	**0.00**	**1.0000**
Simple Effect A @ B2	**1**	**32.0000000**	**32.0000000**	**6.55**	**0.0197**
Simple Effect A @ B3	**1**	**2.0000000**	**2.0000000**	**0.41**	**0.5305**
Simple Effect B @ A1	**2**	**56.0000000**	**28.0000000**	**5.73**	**0.0119**
Simple Effect B @ A2	**2**	**96.0000000**	**48.0000000**	**9.82**	**0.0013**

The Sum of the SS over each Slicing, Sums to the SS for the sliced Main Effect + SS Interaction.
The Sum of the SS for Simple Effects of A @ each B (0+32+2) = 34 = $SS_A + SS_{AxB}$
The Sum of the SS for Simple Effects of B @ each A (56+96) = 152 = $SS_B + SS_{AxB}$

Regression Approach: X_1 = effect code for A membership (A1, X_1 = 1)(A2, X_1 =-1);
X_2 = contrast code for B membership (B1, X_2 = 2) (B2, X_2 = -1) (B3, X_2 = -1);
X_3 = contrast codes for B membership (B1, X_3 = 0) (B2, X_3 = 1) (B3, X_3 = -1);
X_4 = A*B interaction term =(X_1*X_2) X_5 = A*B interaction term =(X_1*X_3)

Output from:

```
proc reg;model y = x1-x5/ stb clb scorr2;
MainA:test x1=0;
MainB:test x2=0, x3=0;
AxB:test x4=0,x5=0;run;
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	158.00000	31.60000	6.46	0.0013
Error	18	88.00000	4.88889		
Corrected Total	23	246.00000			

Root MSE	2.21108	R-Square	0.6423	R^2 = 158/246 = .642
Dependent Mean	8.50000	Adj R-Sq	0.5429	
Coeff Var	26.01274			

Parameter Estimates

Var	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits		Standardized Estimate	Squared SemiPartial Corr Type II	
Int	**1**	**8.50000**	**0.45134**	**18.83**	**<.0001**	**7.55178**	**9.44822**	**0**	**.**	
x1	**1**	**0.50000**	**0.45134**	**1.11**	**0.2825**	**-0.44822**	**1.44822**	**0.15617**	**0.02439**	
x2	**1**	**-1.25000**	**0.31914**	**-3.92**	**0.0010**	**-1.92049**	**-0.57951**	**-0.55216**	**0.30488**	
x3	**1**	**-1.75000**	**0.55277**	**-3.17**	**0.0053**	**-2.91133**	**-0.58867**	**-0.44630**	**0.19919**	**0.5041**
x4	**1**	**-0.25000**	**0.31914**	**-0.78**	**0.4436**	**-0.92049**	**0.42049**	**-0.11043**	**0.01220**	
x5	**1**	**1.25000**	**0.55277**	**2.26**	**0.0364**	**0.08867**	**2.41133**	**0.31879**	**0.10163**	**0.1138**

Test **MainA** Results for Dependent Variable y

```
MainA:test x1=0;
```

Source	DF	Mean Square	F Value	Pr > F
Numerator	**1**	**6.00000**	**1.23**	**0.2825**
Denominator	18	4.88889		

Test **MainB** Results for Dependent Variable y

```
MainB:test x2=0, x3=0;
```

Source	DF	Mean Square	F Value	Pr > F
Numerator	**2**	**62.00000**	**12.68**	**0.0004**
Denominator	18	4.88889		

Test **AxB** Results for Dependent Variable y

```
AxB:test x4=0,x5=0;run;
```

Source	DF	Mean Square	F Value	Pr > F
Numerator	**2**	**14.00000**	**2.86**	**0.0832**
Denominator	18	4.88889		

Model

Parameter	Y_i =	α +	$\beta_1 X_{1i}$	+ $\beta_2 X_{2i}$	+ $\beta_3 X_{3i}$	+ $\beta_4 X_{4i}$	+ $\beta_5 X_{5i}$	+ ε_i	
Statistical	Y_i =	a +	$b_1 X_{1i}$	+ $b_2 X_{2i}$	+ $b_3 X_{3i}$	+ $b_4 X_{4i}$	+ $b_5 X_{5i}$	+ e_i	$e_i = (Y_i - \hat{Y})$
	9 =	8.5 +	.5(1)	-1.25(2)	-1.75(0)	-.25(2)	+1.25(0)	+ 3	3=(9-6)
	7 =	8.5 +	.5(1)	-1.25(2)	-1.75(0)	-.25(2)	+1.25(0)	+ 1	1=(7-6)
$\hat{Y} = \bar{Y}_{11} = 6$	5 =	8.5 +	.5(1)	-1.25(2)	-1.75(0)	-.25(2)	+1.25(0)	- 1	-1=(5-6)
	3 =	8.5 +	.5(1)	-1.25(2)	-1.75(0)	-.25(2)	+1.25(0)	- 3	-3=(9-6)
	12 =	8.5 +	.5(1)	-1.25(-1)	-1.75(1)	-.25(-1)	+1.25(1)	+ 2	2=(12-10)
	12 =	8.5 +	.5(1)	-1.25(-1)	-1.75(1)	-.25(-1)	+1.25(1)	+ 2	2=(12-10)
$\hat{Y} = \bar{Y}_{12} = 10$	11 =	8.5 +	.5(1)	-1.25(-1)	-1.75(1)	-.25(-1)	+1.25(1)	+ 1	1=(11-10)
	5 =	8.5 +	.5(1)	-1.25(-1)	-1.75(1)	-.25(-1)	+1.25(1)	- 5	-5=(5-10)
	13 =	8.5 +	.5(1)	-1.25(-1)	-1.75(-1)	-.25(-1)	+1.25(-1)	+ 2	2=(13-11)
	12 =	8.5 +	.5(1)	-1.25(-1)	-1.75(-1)	-.25(-1)	+1.25(-1)	+ 1	1=(12-11)
$\hat{Y} = \bar{Y}_{13} = 11$	10 =	8.5 +	.5(1)	-1.25(-1)	-1.75(-1)	-.25(-1)	+1.25(-1)	- 1	-1=(10-11)
	9 =	8.5 +	.5(1)	-1.25(-1)	-1.75(-1)	- .5(-1)	+1.25(-1)	- 2	-2=(9-11)
	7 =	8.5 +	.5(-1)	-1.25(2)	-1.75(0)	-.25(-2)	+1.25(0)	+ 1	1=(7-6)
	7 =	8.5 +	.5(-1)	-1.25(2)	-1.75(0)	-.25(-2)	+1.25(0)	+ 1	1=(7-6)
$\hat{Y} = \bar{Y}_{21} = 6$	5 =	8.5 +	.5(-1)	-1.25(2)	-1.75(0)	-.25(-2)	+1.25(0)	- 1	-1=(7-6)
	5 =	8.5 +	.5(-1)	-1.25(2)	-1.75(0)	-.25(-2)	+1.25(0)	- 1	-1=(7-6)
	8 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(1)	-.25(1)	+1.25(-1)	+ 2	2=(8-6)
	7 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(1)	-.25(1)	+1.25(-1)	+ 1	1=(7-6)
$\hat{Y} = \bar{Y}_{22} = 6$	5 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(1)	-.25(1)	+1.25(-1)	- 1	-1=(5-6)
	4 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(1)	-.25(1)	+1.25(-1)	- 2	-2=(4-6)
	14 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(-1)	-.25(1)	+1.25(1)	+ 2	2=(14-12)
	13 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(-1)	-.25(1)	+1.25(1)	+ 1	1=(13-12)
$\hat{Y} = \bar{Y}_{23} = 12$	11 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(-1)	-.25(1)	+1.25(1)	- 1	-1=(11-12)
	10 =	8.5 +	.5(-1)	-1.25(-1)	-1.75(-1)	-.25(1)	+1.25(1)	- 2	-2=(10-12)
			$\Sigma b_1^2 X_1^2=6$	$\Sigma b_2^2 X_2^2=75$	$\Sigma b_3^2 X_3^2=49$	$\Sigma b_4^2 X_4^2=3$	$\Sigma b_5^2 X_5^2=25$		$\Sigma e^2_i = 88$

$SS_A = 6$ $SS_B = 75 + 49 = 124$ $SS_{A*B} = 3 + 25 = 28$