This is an Exploratory Factor Analysis of data related to Test Anxiety.

										Perform	Affect	Worry
	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**	**X9**	fac1_obl	fac2_obl	fac3_obl
1	48	14	22	43	4	14	29	46	8	-.1921	-1.7339	-.8131
2	38	37	30	29	26	35	68	49	16	-.4218	.5282	-.5926
3	57	44	20	46	32	85	75	73	61	-.2308	1.5117	2.7681
4	52	55	25	35	10	49	61	49	44	-.7897	.6686	1.1208
5	40	44	21	49	36	53	84	60	25	-.0856	1.5390	.0940
6	48	21	53	70	38	39	66	63	16	1.2973	.3201	-.5092
7	62	39	29	44	38	32	50	57	29	-.0098	.2718	.6114
8	43	38	17	36	41	24	80	40	19	-.5751	.9811	-.6073
9	24	42	19	40	44	56	80	34	38	-.9367	1.8886	.1827
10	43	16	51	38	26	14	51	42	7	.3172	-.7264	-1.3343
11	55	14	55	65	34	52	62	72	27	1.3590	-.0003	.4844
12	36	12	4	23	30	35	2	33	30	-1.4940	-1.2481	.5856
13	31	4	17	42	22	20	14	17	26	-.9986	-1.4936	-.4178
14	73	25	39	38	25	26	43	68	23	.4536	-.8663	.6386
15	40	13	54	55	46	58	34	65	25	.8407	-.0151	.3689
16	61	17	47	63	26	43	42	68	28	1.0141	-.6360	.6653
17	55	32	60	63	34	56	86	74	29	1.3683	1.0226	.4082
18	78	31	30	39	38	44	71	83	31	.5807	.2768	1.4394
19	44	36	26	44	34	36	43	46	26	-.3756	.2597	.0913
20	48	41	8	36	34	29	67	49	20	-.6864	.6613	-.1331
21	2	29	19	37	29	30	52	22	2	-1.0504	.5253	-2.2037
22	40	28	48	79	29	33	64	59	6	1.2511	.3383	-1.3039
23	52	21	39	60	31	43	55	74	13	.9876	-.1055	-.1249
24	49	29	47	55	30	31	60	49	23	.5002	.1511	-.3220
25	49	15	17	35	27	27	15	41	27	-.7159	-1.2428	.4206
26	68	12	58	52	2	24	53	76	8	1.4291	-1.5881	-.3963
27	12	32	5	2	6	2	29	2	4	-2.4197	-.8877	-2.1248
28	60	22	40	54	34	50	62	76	26	.8893	.1086	.7370
28	43	2	2	21	20	22	28	25	32	-1.4662	-1.5562	.3963
30	36	39	48	53	28	48	73	44	32	.1603	1.0471	-.1297

X1 through **X9** are “item parcels,” “testlets,” “miniscales,” or composite variables (i.e., sums of several Likert type items).

X1 is based on statements such as ***“I worry about what my teachers/classmates think of me.”***

X2 is based on statements such as ***“I get nervous around my classmates.”***

X3 is based on statements such as ***“I hate doing homework.”***

X4 is based on statements such as ***“I hate studying for tests.”***

X5 is based on statements such as ***“I never feel relaxed in my classes.”***

X6 is based on statements such as ***“I don’t think I will do well in school.”***

X7 is based on statements such as ***“I get nervous around my teachers.”***

X8 is based on statements such as ***“I hate taking tests.”***

X9 is based on statements such as ***“I am a complete failure in school.”***

SPSS output from the Factor Analysis module.

Correlation Matrix

		X1	X2	X3	X4	X5	X6	X7	X8	X9
Correlation	X1	1.000	-0.159	-0.152	0.433	0.416	-0.171	0.391	0.320	-0.207
	X2	-0.159	1.000	0.488	-0.098	-0.177	0.533	-0.064	-0.107	0.489
	X3	-0.152	0.488	1.000	-0.073	-0.170	0.627	-0.037	-0.096	0.570
	X4	0.433	-0.098	-0.073	1.000	0.477	-0.085	0.473	0.374	-0.141
	X5	0.416	-0.177	-0.170	0.477	1.000	-0.191	0.431	0.353	-0.231
	X6	-0.171	0.533	0.627	-0.085	-0.191	1.000	-0.045	-0.109	0.622
	X7	0.391	-0.064	-0.037	0.473	0.431	-0.045	1.000	0.340	-0.100
	X8	0.320	-0.107	-0.096	0.374	0.353	-0.109	0.340	1.000	-0.144
	X9	-0.207	0.489	0.570	-0.141	-0.231	0.622	-0.100	-0.144	1.000

Communalities

	Initial	Extraction
X1	1.000	.772
X2	1.000	.733
X3	1.000	.859
X4	1.000	.814
X5	1.000	.429
X6	1.000	.822
X7	1.000	.825
X8	1.000	.905
X9	1.000	.910

Extraction Method: Principal Components Analysis.

I chose analyze the Correlation matrix, instead of the covariance matrix, and chose a **Principal Components Analysis**, instead of Principal Axis Factoring, in order to demonstrate some statistical properties.

Three Factors were extracted based on the eigenvalues in context with theoretical expectations.

$\Sigma\lambda = \Sigma h^2_{initial} = 9$ $\Sigma h^2_{ext} = 7.071$ (7.071/9)x100=78.56% of the variance among the 9 variables is is explained by a three-factor solution.

The **Initial** (or prior) communalities are the diagonals of the matrix analyzed. In a PCA of the Correlation matrix (**R**), all the diagonal entries (i.e., initial communalities) are 1.

The **Extraction** (or posterior) communalities (h^2) are the proportion of each variable explained by the factor structure. Thus, it is an R^2 for how well the factor structure explains each variable.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	3.834	42.599	42.599	3.834	42.599	42.599	3.120
2	2.022	22.469	65.068	2.022	22.469	65.068	2.521
3	1.215	13.496	78.563	1.215	13.496	78.563	2.195
4	.917	10.187	88.750				
5	.499	5.543	94.293				
6	.187	2.079	96.372				
7	.176	1.957	98.329				
8	.150	1.671	100.000				
9	3.293E-16	3.659E-15	100.000				

$\Sigma\lambda = 9$ $\Sigma\lambda = 7.071$ (7.071/9) = .7856

Extraction Method: Principal Components Analysis.

a. When components are correlated, sum of squared loadings cannot be added to obtain a total variance.

Because 1s are used in the diagonal of a PCA, the trace of the matrix equals the number of variables (9 in this case). Since the Sum of the Eigenvalues equals the Trace of the matrix. the Sum of the Eigenvalues equals the number of variables.

Component Matrix[a] (U)

	Component		
	1	2	3
X1	.601	-.351	.536
X2	.327	.727	-.312
X3	.646	-.611	-.260
X4	.750	-.438	-.244
X5	.538	.333	-.169
X6	.801	.394	.160
X7	.697	.323	-.485
X8	.879	-.318	.179
X9	.444	.576	.618

Pattern Matrix[b] (P)

	Component		
	1	2	3
X1	**.533**	-.306	**.622**
X2	-.253	**.842**	.033
X3	**.943**	.020	-.176
X4	**.879**	.176	-.074
X5	.146	**.574**	.133
X6	.197	**.532**	**.554**
X7	.332	**.831**	-.105
X8	**.774**	.059	**.401**
X9	-.282	.221	**.900**

Structure Matrix (S)

	Component		
	1	2	3
X1	**.617**	-.109	**.660**
X2	-.148	**.819**	.166
X3	**.911**	.093	.011
X4	**.885**	.264	.134
X5	.240	**.620**	.285
X6	.367	**.674**	**.707**
X7	**.410**	**.847**	.138
X8	**.859**	.237	**.564**
X9	-.081	.381	**.893**

a. Extraction Method: Principal Components Analysis.
b. Rotation Method: Oblimin with Kaiser Normalization.
a. 3 components extracted
b. Rotation converged in 14 iterations.

Component Correlation Matrix

Component	1	2	3
1	1.000	.118	.194
2	.118	1.000	.215
3	.194	.215	1.000

Interpreting the Factor Structure

The Structure coefficients are standardized regression weights. The Pattern coefficients are Zero-order (bivariate) correlations between each variable and the Factor Scores. Both the Structure and Pattern Matrices lead to similar interpretations.

Factor 1 seems to primarily influence X3 (p_{31}=.943, s_{31}=.911), X4 (p_{41}=.879, s_{41}=.885), and X8 (p_{81}=.774, s_{81}=.859). Also, it influences X1 (p_{11}=.533, s_{11}=.617) to a lesser extent. Given the nature of these variables Factor 1 could be named: **Performance Anxiety**.

Factor 2 seems to primarily influence X2 (p_{22}=.842, s_{22}=.819), X7 (p_{72}=.831, s_{72}=.847), and X5 (p_{52}=.574, s_{52}=.620). Also, Factor 2 influences X6 (p_{62}=.532, s_{62}=.674) to a lesser extent when considering that Factor 3 also has a good deal of influence X6. Given the nature of these variables Factor 2 could be named: **Affective Anxiety**.

Factor 3 seems to primarily influence X9 (p_{93}=.900, s_{93}=.893). But, Factor 3 also influences X1 (p_{13}=.622, s_{13}=.660) and X8 (p_{83}=.401, s_{83}=.564) to a lesser extent when considering that Factor 1 also has a good deal of influence on these two variables. Furthermore, Factor 3 also has some influence on X6 (p_{63}=.554, s_{63}=.707), but Factor 2 also has a influence on X6. Given the nature of these variables Factor 3 could be named: **Cognitive Worry**.

Component Score Coefficient Matrix (B)

	Component		
	1	2	3
X1	.173	.-215	.376
X2	-.120	.404	-.040
X3	.343	-.007	-.148
X4	.311	.060	-.098
X5	.027	.256	.025
X6	.028	.198	.279
X7	.095	.391	-.148
X8	.257	-.032	.202
X9	-.147	.037	.533

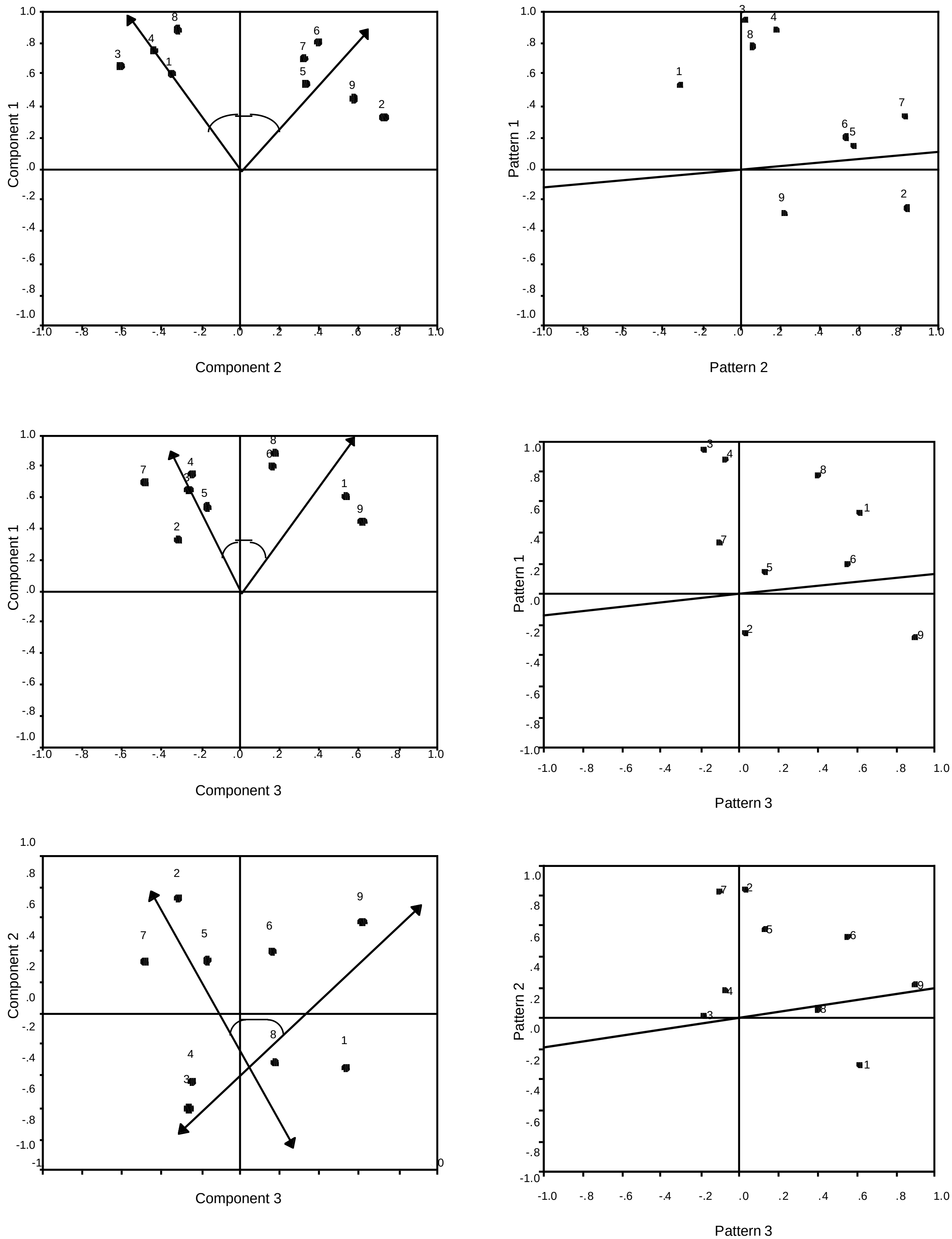
One reason that so many researchers choose orthogonal rotations is that they find it confusing when variables “split” across factors (i.e., have salient loadings on more than one factor). However, this is to be expected if the factors are correlated. Multiple Latent Factors may “cause” a subject to respond (behave, perform) as they do.

These are weights applied to the standardized variables in order to create Factor Scores. For example, to create Factor scores for the 3 factors (fac1_obl, fac2_obl, fac3_obl in the data set above), you would convert the variables to z-scores and compute:

fac1_obl = .173(z1) -.120(z2) +.343(z3) +.311(z4) +.027(z5) +.028(z6) +.095(z7) +.257(z8) -.147(z9)
fac2_obl = -.215(z1) +.404(z2) -.007(z3) +.060(z4) +.256(z5) +.198(z6) +.391(z7) -.032(z8) +.037(z9)
fac3_obl = .376(z1) -.040(z2) -.148(z3) -.098(z4) +.025(z5) +.279(z6) -.148(z7) +.202(z8) +.533(z9)

In matrix notation, the matrix of Factor score, $\mathbf{M}_{30,3,} = \mathbf{Z}_{30,9}\mathbf{B}_{9,3}$

Fortunately, SPSS software computes these automatically and will save them if requested.

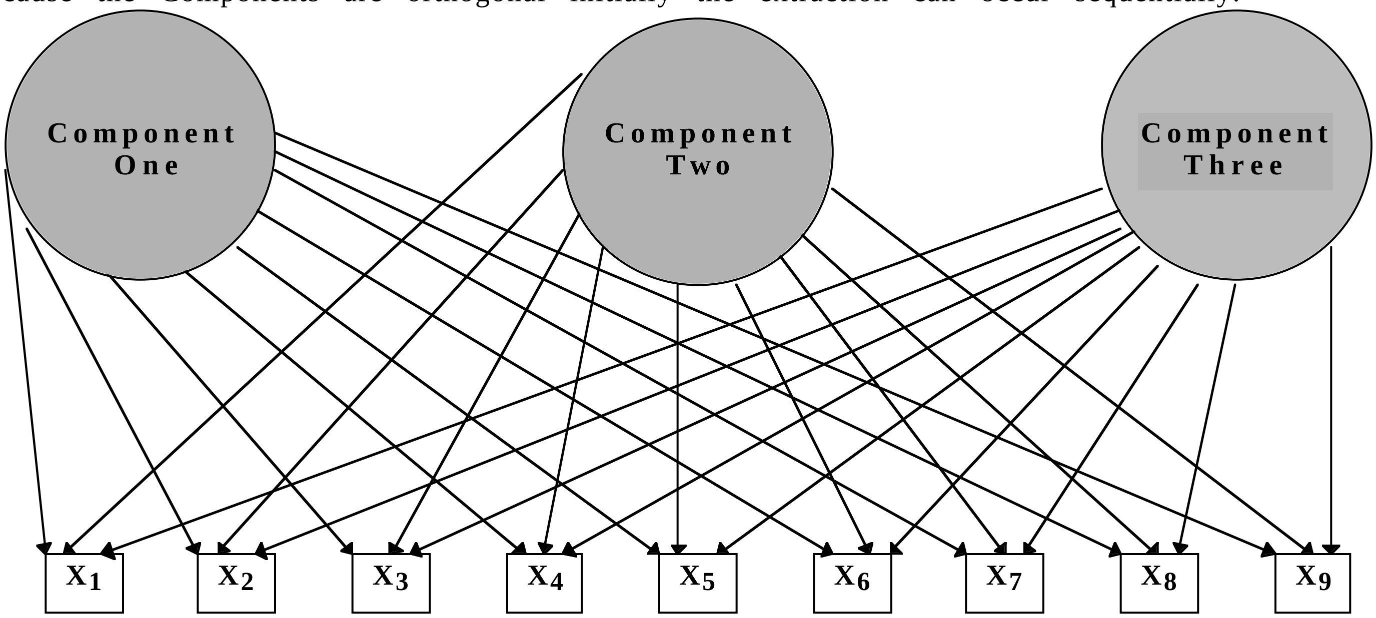

Component 1
Component 2
Pattern 1
Pattern 2
Component 1
Component 3
Pattern 1
Pattern 3
Component 2
Component 3
Pattern 2
Pattern 3

The initial two-dimensional solution using Exploratory Factor Analysis is solving the following model:

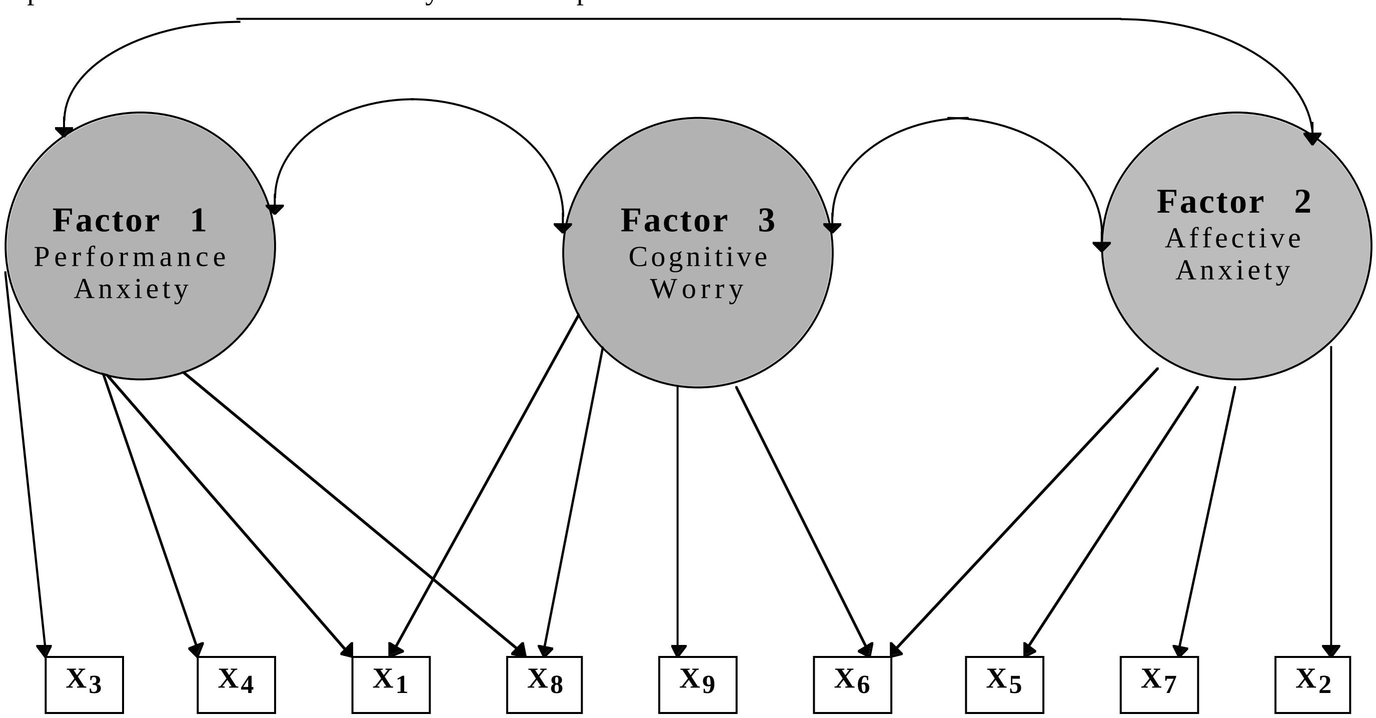
$$c_1 = b_{11}X_1 + b_{12}X_2 + b_{13}X_3 + b_{14}X_4 + b_{15}X_5 + b_{16}X_6 + b_{17}X_7 + b_{18}X_8 + b_{19}X_9$$
$$c_2 = b_{21}X_1 + b_{22}X_2 + b_{23}X_3 + b_{24}X_4 + b_{25}X_5 + b_{26}X_6 + b_{27}X_7 + b_{28}X_8 + b_{29}X_9$$
$$c_3 = b_{31}X_1 + b_{32}X_2 + b_{33}X_3 + b_{34}X_4 + b_{35}X_5 + b_{36}X_6 + b_{37}X_7 + b_{38}X_8 + b_{39}X_9$$

In matrix notation: $\mathbf{C} = \mathbf{B}\,\mathbf{X}$

Because the Components are orthogonal initially the extraction can occur sequentially.



Oblimin Rotation allows the Factors to be correlated, but attempts to keep structure orthogonal Factor correlations of zero) when the Delta parameter is set to 0. It is also an attempt to simplify the interpretation of the solution by rearranging the loadings, maximizing some weights (moving them toward 1) while also minimizing others (moving them toward 0). Although the loadings are technically not zero, if they are near zero the loading can be "ignored for the purposes of interpretation." The solution may be interpreted as follows:



h^2=(.533)(.617)+(-.306)(-.109)+(.622)(.660)=**.772**

Pattern Matrix (P) / **Structure Matrix (S)**

	Pattern Component 1	Pattern Component 2	Pattern Component 3	Structure Component 1	Structure Component 2	Structure Component 3	ps 1	ps 2	ps 3	h^2
X1	**.533**	**-.306**	**.622**	**.617**	**-.109**	**.660**	.329	.033	.410	**.772**
X2	-.253	.842	.033	-.148	.819	.166	.037	.689	.005	.733
X3	.943	.020	-.176	.911	.093	.011	.859	.002	-.002	.859
X4	.879	.176	-.074	.885	.264	.134	.778	.046	-.010	.814
X5	.146	.574	.133	.240	.620	.285	.035	.356	.038	.429
X6	.197	.532	.554	.367	.674	.707	.072	.358	.392	.822
X7	.332	.831	-.105	.410	.847	.138	.136	.704	-.015	.825
X8	.774	.059	.401	.859	.237	.564	.665	.014	.226	.905
X9	**-.282**	**.221**	**.900**	**-.081**	**.381**	**.893**	.023	.084	.803	**.910**

h^2=(-.282)(-.081)+(.221)(.381)+(.900)(.893) = **.910**

Σps = 3.387 + 2.606 + 3.666 = 5.993 = Σh^2

The Pattern coefficients are standardized regression weights and therefore account for the correlation among the 3 factors. The Structure coefficients are zero-order (bivariate) correlations between each of the 9 variables and the 3 sets of Factor Scores.

$X_1 = p_{11}F_1 + p_{12}F_2 + p_{13}F_3$

$X_1 = .533F_1 - .306F_2 + .622F_3$

. . . . . .

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.879a	**.772**	.746	8.1017

a Predictors: (Constant), Perform (F1), Affect (F2), Worry (F3)

Coefficients

	Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound	Correlations Zero-order	Correlations Partial	Correlations Part	Collinearity Statistics Tol.	Collinearity Statistics VIF
1 **(Constant)**	46.233	1.479		31.256	.000	43.193	49.274					
Perform (F1)	8.567	1.538	**.533**	5.569	.000	5.405	11.729	**.617**	.738	.521	.956	1.046
Affect (F2)	-4.918	1.546	**-.306**	-3.182	.004	-8.095	-1.741	**-.109**	-.529	-.298	.948	1.055
Worry (F3)	10.008	1.564	**.622**	6.398	.000	6.793	13.224	**.660**	.782	.598	.925	1.081

a Dependent Variable: X1

. . . . .

$X_9 = p_{91}F_1 + p_{92}F_2 + p_{93}F_3$

$X_9 = -.282F_1 + .221F_2 + .900F_3$

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.954a	**.910**	.900	3.9698

a Predictors: (Constant), Perform (F1), Affect (F2), Worry (F3)

Coefficients

	Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound	Correlations Zero-order	Correlations Partial	Correlations Part	Collinearity Statistics Tol.	Collinearity Statistics VIF
1 **(Constant)**	23.367	.725		32.239	.000	21.877	24.856					
Perform (F1)	-3.534	.754	**-.282**	-4.689	.000	-5.084	-1.985	**-.081**	-.667	-.275	.956	1.046
Affect (F2)	2.770	.757	**.221**	3.658	.001	1.213	4.327	**.381**	.583	.215	.948	1.055
Worry (F3)	11.288	.767	**.900**	14.726	.000	9.712	12.863	**.893**	.865	.598	.925	1.081

a Dependent Variable: X9

From the Fundamental Factor Theorem, the original correlation matrix (R) can be reproduced by $\mathbf{R}_{(r)} = \mathbf{P}_{p,f}\mathbf{F}_{f,f}\mathbf{P}'_{f,p}$, where $\mathbf{P}_{p,f}$ is a Pattern matrix and $\mathbf{F}_{f,f}$ is the Factor correlation matrix.

Pattern Matrix (P) x

	Component		
	1	2	3
X1	.533	-.306	.622
X2	-.253	.842	.033
X3	.943	.020	-.176
X4	.879	.176	-.074
X5	.146	.574	.133
X6	.197	.532	.554
X7	.332	.831	-.105
X8	.774	.059	.401
X9	-.282	.221	.900

Component Correlation Matrix (F)

Component	1	2	3
1	1.000	.118	.194
2	.118	1.000	.215
3	.194	.215	1.000

= Structure Matrix (S)

	Component		
	1	2	3
X1	.617	-.109	.660
X2	-.148	.819	.166
X3	.911	.093	.011
X4	.885	.264	.134
X5	.240	.620	.285
X6	.367	.674	.707
X7	.410	.847	.138
X8	.859	.237	.564
X9	-.081	.381	.893

The Structure Matrix is computed as $\mathbf{S}_{p,f} = \mathbf{P}_{p,f}\mathbf{F}_{f,f}$; therefore, the correlation matrix can also be reproduced as $\mathbf{R}_{(r)} = \mathbf{S}_{p,f}\mathbf{P}'_{f,p}$.

Structure Matrix (S) x Transpose of Pattern Matrix (P′) = $R_{(r)}$

	Component		
	1	2	3
X1	.617	-.109	.660
X2	-.148	.819	.166
X3	.911	.093	.011
X4	.885	.264	.134
X5	.240	.620	.285
X6	.367	.674	.707
X7	.410	.847	.138
X8	.859	.237	.564
X9	-.081	.381	.893

.533	-.253	.943	.879	.146	.197	.332	.774	-.282
-.306	.842	.020	.176	.574	.532	.831	.059	.221
.622	.033	-.176	-.074	.133	.554	-.105	.401	.900

Correlation Matrix

		X1	X2	X3	X4	X5	X6	X7	X8	X9
Reproduced Correlation	X1	.772[b]	-.226	.464	.474	.116	.429	.045	.736	.396
	X2	-.226	.733[b]	-.152	.003	.471	.499	.614	.001	.372
	X3	.464	-.152	.859[b]	.816	.188	.235	.378	.715	-.226
	X4	.474	.003	.816	.814[b]	.299	.389	.499	.755	-.070
	X5	.116	.471	.188	.299	.429[b]	.535	.565	.337	.326
	X6	.429	.499	.235	.389	.535	.822[b]	.608	.607	.682
	X7	.045	.614	.378	.499	.565	.608	.825[b]	.423	.196
	X8	.736	.001	.715	.755	.337	.607	.423	.905[b]	.317
	X9	.396	.372	-.226	-.070	.326	.682	.196	.317	.910[b]
Residual[a]	X1		.171	-.052	-.116	-.080	-.186	.125	.076	-.076
	X2	.171		.012	-.073	-.252	-.117	.064	.070	-.047
	X3	-.052	.012		-.032	-.056	.035	-.026	-.057	.070
	X4	-.116	-.073	-.032		.042	.070	-.093	-.077	.067
	X5	-.080	-.252	-.056	.042		-.001	-.166	-.051	-.023
	X6	-.186	-.117	.035	.070	-.001		-.087	-.026	.038
	X7	.125	.064	-.026	-.093	-.166	-.087		.043	-.022
	X8	.076	.070	-.057	-.077	-.051	-.026	.043		-.087
	X9	-.076	-.047	.070	.067	-.023	.038	-.022	-.087	

a Residuals are computed between observed and reproduced correlations. There are 24 (66.0%) nonredundant
b Reproduced communalities

Reproduced Correlation Matrix: $\mathbf{R}_{(r)9,9} = \mathbf{S}_{9,3}\mathbf{P}'_{3,9}$