Multivariate Analysis of Variance (MANOVA)

Condition	(Y_1)	(Y_2)	(Y_3)
Control A_1	9	9	13
	9	8	14
	8	8	12
	7	8	10
	7	7	11
	$\overline{Y}_{11} = 8$	$\overline{Y}_{12} = 8$	$\overline{Y}_{13} = 12$
Treatment A_2	9	17	7
	8	18	7
	8	16	6
	8	14	5
	7	15	5
	$\overline{Y}_{21} = 8$	$\overline{Y}_{22} = 16$	$\overline{Y}_{23} = 6$
	$\overline{Y}_{*1} = 8$	$\overline{Y}_{*2} = 12$	$\overline{Y}_{*3} = 9$

		Y1	Y2	Y3
Y1	Pearson Correlation	1.000	**.125**	**.320**
	Sig. (2-tailed)	.	.732	.367
	N	10	10	10
Y2	Pearson Correlation	.125	1.000	**-.837****
	Sig. (2-tailed)	.732	.	.003
	N	10	10	10
Y3	Pearson Correlation	.320	-.837**	1.000
	Sig. (2-tailed)	.732	.003	.
	N	10	10	10

MANOVA MODEL: $Y_{ij} = \mu^* + \alpha_j + \varepsilon_{ij}$

Null hypothesis is that the group centroids are identical.

H_0: $\mu_1 = \mu_2$.

In this J=2 group case, H_0: $\mu_{11} = \mu_{21}$

$\mu_{12} = \mu_{22}$

$\mu_{13} = \mu_{23}$.

In the one-way ANOVA, the Total SS are partitioned into Between-Group SS (Explained Variance associated with the Hypothesis of interest) and Within-Group SS (Unexplained Variance attributed to Error).

In the MANOVA, this principle applies, but the Between-Groups SS are generalized to an **H** matrix that contains **Explained Covariance** associated with the **H**ypothesis of interest. The Within-Group SS is generalized to the **E** matrix that contains **Unexplained Covariance** attributed to **E**rror. These two matrices sum to the **T** matrix, the generalization of the Total SS.

Computation of the H Matrix

The k^{th} row , p^{th} column of the **H** matrix is $\Sigma n_j(\overline{Y}_{jp} - \overline{Y}_{*p})(\overline{Y}_{jk} - \overline{Y}_{*k})$

	(Y_1) p =1	(Y_2) p = 2	(Y_3) p =3
(Y_1) k =1	5(8-8)(8-8) +5(8-8)(8-8) = **0**	5(8-8)(8-12) + 5(8-8)(16-12) = **0**	5(8-8)(12-9) + 5(8-8)(6-9) = **0**
(Y_2) k =2	5(8-12)(8-8) + 5(16-12)(8-8) = **0**	5(8-12)(8-12) + 5(16-12)(16-12) = **160**	5(8-12)(12-9) + 5(16-12)(6-9) = **-120**
(Y_3) k =3	5(12-9)(8-8) + 5(6-9)(8-8) = **0**	5(12-9)(8-12) + 5(6-9)(16-12) = **-120**	5(12-9)(12-9) + 5(6-9)(6-9) = **90**

Computation of the E Matrix

The **E** matrix is the sum of the Within-Group Sum of Squares and Cross-Products over the J groups; $\mathbf{E} = \Sigma \mathbf{S}_j$.

The k^{th} row , p^{th} column of the $\mathbf{S}_j = \Sigma(Y_{ip} - \overline{Y}_{jp})(Y_{ik} - \overline{Y}_{jk})$.

For Group 1,

$\mathbf{S}_1$ =	(Y_1) p =1	(Y_2) p = 2	(Y_3) p =3
(Y_1) k =1	(9-8)(9-8) = 1 + (9-8)(9-8) = 1 + (8-8)(8-8) = 0 + (7-8)(7-8) = 1 + (7-8)(7-8) = 1 **4**	(9-8)(9-8) = 1 + (9-8)(8-8) = 0 + (8-8)(8-8) = 0 + (7-8)(8-8) = 0 + (7-8)(7-8)= 1 **2**	(9-8)(13-12)= 1 + (9-8)(14-12) = 2 + (8-8)(12-12) = 0 + (7-8)(10-12) = 2 + (7-8)(11-12) = 1 **6**
(Y_2) k =2	(9-8)(9-8) = 1 + (8-8)(9-8) = 0 + (8-8)(8-8) = 0 + (8-8)(7-8) = 0 + (7-8)(7-8) = 1 **2**	(9-8)(9-8) = 1 + (8-8)(8-8) = 0 + (8-8)(8-8) = 0 + (8-8)(8-8) = 0 + (7-8)(7-8) = 1 **2**	(9-8)(13-12) = 1 + (8-8)(14-12) = 0 + (8-8)(12-12) = 0 + (8-8)(10-12) = 0 + (7-8)(11-12) = 1 **2**
(Y_3) k =3	(13-12)(9-8) = 1 + (14-12)(9-8) = 2 + (12-12)(8-8) = 0 + (10-12)(7-8) = 2 + (11-12)(7-8) = 1 **6**	(13-12)(9-8) = 1 + (14-12)(8-8) = 0 + (12-12)(8-8) = 0 + (10-12)(8-8) = 0 + (11-12)(7-8) = 1 **2**	(13-12)(13-12) = 1 + (14-12)(14-12) = 4 + (12-12)(12-12) = 0 + (10-12)(10-12) = 4 + (11-12)(11-12) = 1 **10**

For Group 2,

$\mathbf{S}_2$ =	(Y_1) p =1	(Y_2) p = 2	(Y_3) p =3
(Y_1) k =1	+ (9-8)(9-8) = 1 + (8-8)(8-8) = 0 + (8-8)(8-8) = 0 + (8-8)(8-8) = 0 + (7-8)(7-8) = 1 **2**	+ (9-8)(17-16) = 1 + (8-8)(18-16) = 0 + (8-8)(16-16) = 0 + (8-8)(14-16) = 0 + (7-8)(15-16) = 1 **2**	+ (9-8)(7 - 6) = 1 + (8-8)(7 - 6) = 0 + (8-8)(6 - 6) = 0 + (8-8)(5 - 6) = 0 + (7-8)(5 - 6) = 1 **2**
(Y_2) k =2	+ (17-16)(9-8) = 1 + (18-16)(8-8) = 0 + (16-16)(8-8) = 0 + (14-16)(8-8) = 0 + (15-16)(7-8) = 1 **2**	+ (17-16)(17-16) = 1 + (18-16)(18-16) = 4 + (16-16)(16-16) = 0 + (14-16)(14-16) = 4 + (15-16)(15-16) = 1 **10**	+ (17-16)(7 - 6) = 1 + (18-16)(7 - 6) = 2 + (16-16)(6 - 6) = 0 + (14-16)(5 - 6) = 2 + (15-16)(5 - 6) = 1 **6**
(Y_3) k =3	+ (7 - 6)(9 - 8) = 1 + (7 - 6)(8 - 8) = 0 + (6 - 6)(8 - 8) = 0 + (5 - 6)(8 - 8) = 0 + (5 - 6)(7 - 8) = 1 **2**	+ (7 - 6)(17-16) = 1 + (7 - 6)(18-16) = 2 + (6 - 6)(16-16) = 0 + (5 - 6)(14-16) = 2 + (5 - 6)(15-16) = 1 **6**	+ (7 - 6)(7 - 6) = 1 + (7 - 6)(7 - 6) = 1 + (6 - 6)(6 - 6) = 0 + (5 - 6)(5 - 6) = 1 + (5 - 6)(5 - 6) = 1 **4**

Thus with $J = 2$ groups $\mathbf{E} = \mathbf{S}_1 + \mathbf{S}_2 = \begin{Bmatrix} 4 & 2 & 6 \\ 2 & 2 & 2 \\ 6 & 2 & 10 \end{Bmatrix} + \begin{Bmatrix} 2 & 2 & 2 \\ 2 & 10 & 6 \\ 2 & 6 & 4 \end{Bmatrix} = \begin{Bmatrix} \mathbf{6} & \mathbf{4} & \mathbf{8} \\ \mathbf{4} & \mathbf{12} & \mathbf{8} \\ \mathbf{8} & \mathbf{8} & \mathbf{14} \end{Bmatrix}$

Computation of the T Matrix

The k^{th} row , p^{th} column of the **T** matrix is $\Sigma(Y_{ip} - \overline{Y}_{*p})(Y_{ik} - \overline{Y}_{*k})$

	(Y_1) p =1	(Y_2) p = 2	(Y_3) p =3
(Y_1) k =1	(9-8)(9-8) + (9-8)(9-8) + (8-8)(8-8) + (7-8)(7-8) + (7-8)(7-8) + (9-8)(9-8) + (8-8)(8-8) + (8-8)(8-8) + (8-8)(8-8) + (7-8)(7-8) = **6**	(9-8)(9-12) + (9-8)(8-12) + (8-8)(8-12) + (7-8)(8-12) + (7-8)(7-12) + (9-8)(17-12) + (8-8)(18-12) + (8-8)(16-12) + (8-8)(14-12) + (7-8)(15-12) = **4**	(9-8)(13-9) + (9-8)(14-9) + (8-8)(12-9) + (7-8)(10-9) + (7-8)(11-9) + (9-8)(7 - 9) + (8-8)(7 - 9) + (8-8)(6 - 9) + (8-8)(5 - 9) + (7-8)(5 - 9) = **8**
(Y_2) k =2	(9-12)(9-8) + (8-12)(9-8) + (8-12)(8-8) + (8-12)(7-8) + (7-12)(7-8) + (17-12)(9-8) + (18-12)(8-8) + (16-12)(8-8) + (14-12)(8-8) + (15-12)(7-8) = **4**	(9-12)(9-12) + (8-12)(8-12) + (8-12)(8-12) + (8-12)(8-12) + (7-12)(7-12) + (17-12)(17-12) + (18-12)(18-12) + (16-12)(16-12) + (14-12)(14-12) + (15-12)(15-12) = **172**	(9-12)(13-9) + (8-12)(14-9) + (8-12)(12-9) + (8-12)(10-9) + (7-12)(11-9) + (17-12)(7 - 9) + (18-12)(7 - 9) + (16-12)(6 - 9) + (14-12)(5 - 9) + (15-12)(5 - 9) = **-112**
(Y_3) k =3	(13-9)(9-8) + (14-9)(9-8) + (12-9)(8-8) + (10-9)(7-8) + (11-9)(7-8) + (7 - 9)(9 - 8) + (7 - 9)(8 - 8) + (6 - 9)(8 - 8) + (5 - 9)(8 - 8) + (5 - 9)(7 - 8) = **8**	(13-9)(9-12) + (14-9)(8-12) + (12-9)(8-12) + (10-9)(8-12) + (11-9)(7-12) + (7 - 9)(17-12) + (7 - 9)(18-12) + (6 - 9)(16-12) + (5 - 9)(14-12) + (5 - 9)(15-12) = **-112**	(13-9)(13-9) + (14-9)(14-9) + (12-9)(12-9) + (10-9)(10-9) + (11-9)(11-9) + (7 - 9)(7 - 9) + (7 - 9)(7 - 9) + (6 - 9)(6 - 9) + (5 - 9)(5 - 9) + (5 - 9)(5 - 9) **104**

Note that **T** = **H** + **E** and that they are all pxp square matrices and thus are conformable for addition and multiplication.

Multivariate Test Statistics

The major MANOVA test statistics can be based on taking the first s eigenvalues (λ) from various matrices, where s is the smallest of the df_h and p.

In the one-way design, $s = \min[(J\text{-}1), p]$. Only the first s eigenvalues are used because all other will be zero or below. However, there are occasions where these matrices are not symmetric or indeterminant (not invertable), and thus, a generalized eigenvalue solution must be used.

1. Wilks' Lambda (W) can be computed from the non-zero eigenvalues of the **E** matrix multiplied by the inverse of the **T** matrix. That is, W can be computed from the non-zero eigenvalues of $\mathbf{ET}^{-1}$. Since a determinant is equal to the product of eigenvalues, Wilks lambda is equal to the product of eigenvalues of the matrix $\mathbf{ET}^{-1}$: $W = \pi\lambda_{\mathbf{ET}^{-1}}$.
Because the matrix may not be symmetric a generalized eigenvalue solution must be computed, W can be computed in a simpler fashion. Wilks' Lambda is the ratio of determinants for the **E** and **T** matrices, $W = |\mathbf{E}|/|\mathbf{T}| = |\mathbf{E}|/|(\mathbf{H}+\mathbf{E})|$. Thus, as the Explained covariation (**H**) becomes large, the Error variation (**E**) is reduced and the W statistic becomes small and approaches zero. If there were only $p = 1$ variable, $W = 1 - \text{eta}^2$. Thus, W is the multivariate generalization of $1 - R^2$ or $1 - \text{eta}^2$.

2. Hotelling's Trace is the multivariate generalization of the F-ratio. Hotelling's Trace (T) can be computed from the eigenvalues of the **H** matrix multiplied by the inverse of the **E** matrix. That is, T can be computed from the first s (non-zero) eigenvalues of $\mathbf{HE}^{-1}$. Hotelling's Trace is equal to the sum of first s (non-zero) eigenvalues of the $\mathbf{HE}^{-1}$ matrix: $T = \Sigma\lambda_{\mathbf{HE}^{-1}}$.
Another formulation that illustrates T as the generalization of the F-ratio will be demonstrated later.

3. Roy's Largest Root (R) is also computed from the eigenvalues of the **H** matrix multiplied by the inverse of the **E** matrix. However, R is equal to the first (and largest) eigenvalue of the $\mathbf{HE}^{-1}$ matrix: $R = \max[\lambda_{\mathbf{HE}^{-1}}]$.

3. Pillai's Trace is the multivariate generalization of eta^2 or R^2. Pillai's Trace (V) can be computed from the eigenvalues of the **H** matrix multiplied by the inverse of the **T** matrix. That is, V can be computed from the first s (non-zero) eigenvalues of $\mathbf{HT}^{-1}$. Pillai's Trace is equal to the sum of first s (non-zero) eigenvalues of the $\mathbf{HT}^{-1}$ matrix: $V = \Sigma\lambda_{\mathbf{HT}^{-1}}$. Another formulation that illustrates V as the generalization of eta^2 (or R^2) will be demonstrated later.

Choice among MANOVA Test Statistics

As was the case with the univariate ANOVA, there is an assumption of *Homogeneity of Covariance*. for MANOVA test statistics. That is in this case, $\mathbf{S}_1$ and $\mathbf{S}_2$ would be assumed to be equal in the population, which may not be the case.

Olson (1976, 1979) suggests the Pillai-Bartlett trace (V) as an omnibus MANOVA test statistic for its superior robustness to heterogeneous variances. Stevens (1979, 1980) contends that the robustness of V, Wilks' lambda (W) and the Hotelling trace (T) are similar and that their power functions are highly sensitive to slight covariance inequalities. Sheehan and Beasley (1994) demonstrated that violations of the assumption of homogeneous variance-covariance matrices in the form of covariance inequalities does not affect the robustness of V, W, or T, while T is slightly more powerful under such conditions. Under conditions of diffuse noncentrality structures, V is a clear choice.

Hotelling's Trace (*T*) as a Generalization of the univariate *F*-ratio

In calculating a univariate *F*-ratio for say variable Y_3, we would take the SS_H and divide it by SS_E then multiply it times the ratio of *df*s.

Thus, $F(1, 8) = (90/14)(8/1) = 51.43$

For a multivariate statistic, the inverse of **E** must be computed.

Given that $\mathbf{E} = \begin{Bmatrix} 6 & 4 & 8 \\ 4 & 12 & 8 \\ 8 & 8 & 14 \end{Bmatrix}$ $\quad \mathbf{E}^{-1} = \begin{Bmatrix} 0.7222 & 0.0556 & -0.4444 \\ -0.0556 & 0.1389 & -0.1111 \\ -0.4444 & -0.1111 & 0.3889 \end{Bmatrix}$

Hotelling's Trace is equal to $T = \text{trace}(\mathbf{HE}^{-1})$; however, to demonstrates its relationship to the univariate *F*, Hotelling's Trace can be expressed as:

$$T = \Sigma n_j\, \mathbf{d}_j'\mathbf{E}^{-1}\mathbf{d}_j \text{ , where } \mathbf{d}_j' = \text{a deviation vector for the } j\text{th group.}$$

That is, $\mathbf{d}_j' = \{(\overline{Y}_{j1}-\overline{Y}_{*1}), (\overline{Y}_{j2}-\overline{Y}_{*2}) \ldots (\overline{Y}_{jp}-\overline{Y}_{*p})\}$

Thus, $\mathbf{d}_1' = (8\text{-}8)\ (8\text{-}12)\ (12\text{-}9) = \{0\ \ -4\ \ 3\}$

and $\mathbf{d}_2' = (8\text{-}8)\ (16\text{-}12)\ (6\text{-}9) = \{0\ \ 4\ \ -3\}$.

Note that $\Sigma n_j\, \mathbf{d}_j\, \mathbf{d}_j' = \mathbf{H}$.

That is, $n_1\mathbf{d}_1\,\mathbf{d}_1' = 5 \begin{Bmatrix} 0 \\ 4 \\ -3 \end{Bmatrix} \{0\ \ -4\ \ 3\} = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & 80 & -60 \\ 0 & -60 & 45 \end{Bmatrix}$ and $n_2\mathbf{d}_2\mathbf{d}_2' = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & 80 & -60 \\ 0 & -60 & 45 \end{Bmatrix}$

To calculate Hotelling Trace, then for Group 1,

$\mathbf{d}_1' = \{0\ \ -4\ \ 3\} \begin{Bmatrix} 0.7222 & 0.0556 & -0.4444 \\ -0.0556 & 0.1389 & -0.1111 \\ -0.4444 & -0.1111 & 0.3889 \end{Bmatrix} = \{-1.1111\ \ -0.8888\ \ 1.6111\} \begin{Bmatrix} 0 \\ -4 \\ 3 \end{Bmatrix} = 8.3888$

and for Group 2,

$\mathbf{d}_2' = \{0\ \ 4\ \ -3\} \begin{Bmatrix} 0.7222 & 0.0556 & -0.4444 \\ -0.0556 & 0.1389 & -0.1111 \\ -0.4444 & -0.1111 & 0.3889 \end{Bmatrix} = \{1.1111\ \ 0.8888\ \ -1.6111\} \begin{Bmatrix} 0 \\ 4 \\ -3 \end{Bmatrix} = 8.3888$

Thus, the Hotelling Trace is equal to $T = \Sigma n_j\, \mathbf{d}_j'\mathbf{E}^{-1}\mathbf{d}_j = 5(8.3888) + 5(8.3888) = 83.8888$

Thus, the Hotelling Trace can be viewed as a multivariate generalization of the *F* test.

Pillai's Trace (*V*) as a Generalization of the eta² (or R²)

In calculating a univariate eta-Squared for variable Y_3, we would take the SS_H and divide it by SS_T; thus eta-squared = 90/140 = .865.

Again for a multivariate statistic, the inverse of **T** must be computed.

Given that $\mathbf{T} = \begin{Bmatrix} 6 & 4 & 8 \\ 4 & 172 & -112 \\ 8 & -112 & 104 \end{Bmatrix}$ $\quad \mathbf{T}^{-1} = \begin{Bmatrix} 0.4372 & -0.1073 & -0.1492 \\ -0.1073 & 0.0458 & 0.0576 \\ -0.1492 & 0.0576 & 0.0831 \end{Bmatrix}$

The Pillai Trace is equal to $V = \text{trace}(\mathbf{HT}^{-1})$

To calculate the Pillai Trace, then:

$\mathbf{HT}^{-1} = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & 160 & -120 \\ 0 & -120 & 90 \end{Bmatrix} \begin{Bmatrix} 0.4372 & -0.1073 & -0.1492 \\ -0.1073 & 0.0458 & 0.0576 \\ -0.1492 & 0.0576 & 0.0831 \end{Bmatrix} = \begin{Bmatrix} \boxed{\mathbf{0}} & 0 & 0 \\ 0.733 & \boxed{\mathbf{.419}} & -0.759 \\ -0.550 & -0.314 & \boxed{\mathbf{.569}} \end{Bmatrix}$

Thus, the Pillai Trace is equal to $V = \text{trace}(\mathbf{HT}^{-1}) = 0 + 0.419 + 0.569 = 0.988$.

Thus, the Pillai Trace can be viewed as a multivariate generalization of eta².

General Linear Model

Descriptive Statistics

	FACTORA	Mean	Std. Deviation	N
Y1	Control	8.0000	1.0000	5
	Treatment	8.0000	.7071	5
	Total	8.0000	.8165	10
Y2	Control	8.0000	.7071	5
	Treatment	16.0000	1.5811	5
	Total	12.0000	4.3716	10
Y3	Control	12.0000	1.5811	5
	Treatment	6.0000	1.0000	5
	Total	9.0000	3.3993	10

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.	Eta Squared
Intercept	Pillai's Trace	.995	407.778[a]	3.000	6.000	.000	.995
	Wilks' Lambda	.005	407.778[a]	3.000	6.000	.000	.995
	Hotelling's Trace	203.889	407.778[a]	3.000	6.000	.000	.995
	Roy's Largest Root	203.889	407.778[a]	3.000	6.000	.000	.995
FACTORA	Pillai's Trace	.988	167.778[a]	3.000	6.000	.000	.988
	Wilks' Lambda	.012	167.778[a]	3.000	6.000	.000	.988
	Hotelling's Trace	83.889	167.778[a]	3.000	6.000	.000	.988
	Roy's Largest Root	83.889	167.778[a]	3.000	6.000	.000	.988

a. Exact statistic

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Corrected Model	Y1	.000	1	.000	.000	1.000	.000
	Y2	160.000	1	160.000	106.667	.000	.930
	Y3	90.000	1	90.000	51.429	.000	.865
Intercept	Y1	640.000	1	640.000	853.333	.000	.991
	Y2	1440.000	1	1440.000	960.000	.000	.992
	Y3	810.000	1	810.000	462.857	.000	.983
FACTORA	**Y1**	**.000**	**1**	**.000**	**.000**	**1.000**	**.000**
	Y2	**160.000**	**1**	**160.000**	**106.667**	**.000**	**.930**
	Y3	**90.000**	**1**	**90.000**	**51.429**	**.000**	**.865**
Error	Y1	6.000	8	.750			
	Y2	12.000	8	1.500			
	Y3	14.000	8	1.750			
Total	Y1	646.000	10				
	Y2	1612.000	10				
	Y3	914.000	10				
Corrected Total	Y1	6.000	9				
	Y2	172.000	9				
	Y3	104.000	9				

Discriminant Analysis

Group Statistics

		Mean	Std. Deviation	Valid N (listwise) Unweighted	Weighted
Control	**Y1**	8.0000	1.0000	**5**	**5.000**
	Y2	8.0000	.7071	**5**	**5.000**
	Y3	8.0000	.8165	**5**	**5.000**
Treatment	**Y1**	8.0000	.7071	**5**	**5.000**
	Y2	16.0000	1.5811	**5**	**5.000**
	Y3	12.0000	4.3716	**5**	**5.000**
Total	**Y1**	12.0000	1.5811	**10**	**10.000**
	Y2	6.0000	1.0000	**10**	**10.000**
	Y3	9.0000	3.3993	**10**	**10.000**

In the 2-group situation,
Wilks' Lambda (W) is functionally related to the univariate Eta Squared, η^2.
$\eta^2 = = (1 - W) = (1-.070) = .930$

Tests of Equality of Group Means

	Wilks' Lambda	F	df1	df2	Sig.
Y1	**1.000**	**.000**	**1**	**8**	**1.000**
Y2	**.070**	**106.667**	**1**	**8**	**.000**
Y3	**.135**	**51.429**	**1**	**8**	**.000**

In the 2-group situation,
Wilks' Lambda (W) is functionally related to Pillai's Trace (V).
$V = (1 - W) = (1-.012) = .998$
Which is also related to the Canonical Correlation coefficient
$Rc = \sqrt{V} = \sqrt{1 - W} = \sqrt{.998} = .994$

Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	**83.889[a]**	**100. 0**	**100.0**	**.994**

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-Square	df	Sig
1	**.012**	**28.869**	**3**	**.000**

With 2 groups Roy's Root & Hotelling Trace are identical.

Standardized Canonical Discriminant Function Coefficients

	Function 1
Y1	**1.316**
Y2	**1.063**
Y3	**-2.081**

Standardized Coefficients used to evaluate the Relative Contribution of each variable to the Discriminant Function that separate the groups

Structure Matrix

	Function 1
Y1	**.000**
Y2	**.399**
Y3	**-.277**

Pooled within-group correlations between discriminating variables and standardized canonical discriminant functions

Computed as $S = \dfrac{\sum (Y - \bar{Y}_j)(F - \bar{F}_j)}{\sqrt{\sum (Y - \bar{Y}_j)^2}\sqrt{\sum (F - \bar{F}_j)^2}}$

Canonical Discriminant Function Coefficients

	Function 1
Y1	**1.519**
Y2	**.868**
Y3	**-1.573**
(Constant)	**-8.409**

Unstandardized coefficients

Functions at Group Centroids

Finish	Function 1
Control	**-8.192**
Treatment	**8.192**

General Linear Model

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.	Eta Squared
Intercept	Pillai's Trace	.999	2463.997[a]	11.000	27.000	.000	.999
	Wilks' Lambda	.001	2463.997[a]	11.000	27.000	.000	.999
	Hotelling's Trace	1003.851	2463.997[a]	11.000	27.000	.000	.999
	Roy's Largest Root	1003.851	2463.997[a]	11.000	27.000	.000	.999
DAMAGE	Pillai's Trace	1.138	3.363	22.000	56.000	.000	.569
	Wilks' Lambda	.047	8.839[a]	22.000	54.000	.000	.783
	Hotelling's Trace	16.241	19.194	22.000	52.000	.000	.890
	Roy's Largest Root	15.995	40.715[b]	11.000	28.000	.000	.941

a. Exact statistic

Group Statistics

DAMAGE		Mean	Std. Deviation	Valid N (listwise) Unweighted	Valid N (listwise) Weighted
BD LEFT	INFO	78.900	10.5351	10	10.000
	SIMIL	88.200	12.8996	10	10.000
	ARITH	82.500	16.3860	10	10.000
	VOCAB	79.700	12.8413	10	10.000
	COMPRE	83.900	8.3060	10	10.000
	DIGSPAN	84.800	11.7170	10	10.000
	PICTCOMP	84.400	12.9889	10	10.000
	PICTARR	81.700	15.7131	10	10.000
	BLOCKDES	92.800	7.1926	10	10.000
	OBJASSEM	91.300	13.6060	10	10.000
	CODING	88.500	15.4074	10	10.000
BD DIFFUSE	INFO	96.400	9.8162	20	20.000
	SIMIL	94.800	13.3480	20	20.000
	ARITH	98.950	12.4202	20	20.000
	VOCAB	100.000	8.5594	20	20.000
	COMPRE	101.350	7.6315	20	20.000
	DIGSPAN	101.550	12.5634	20	20.000
	PICTCOMP	99.550	15.5444	20	20.000
	PICTARR	96.700	13.4755	20	20.000
	BLOCKDES	104.950	15.2090	20	20.000
	OBJASSEM	98.850	12.4827	20	20.000
	CODING	101.850	11.3892	20	20.000
BD RIGHT	INFO	117.700	6.1833	10	10.000
	SIMIL	113.600	11.2665	10	10.000
	ARITH	107.700	14.5453	10	10.000
	VOCAB	122.000	9.4163	10	10.000
	COMPRE	121.500	9.4192	10	10.000
	DIGSPAN	112.100	6.8060	10	10.000
	PICTCOMP	103.700	8.9944	10	10.000
	PICTARR	107.500	12.5632	10	10.000
	BLOCKDES	104.000	9.0062	10	10.000
	OBJASSEM	104.600	6.7528	10	10.000
	CODING	108.100	18.0644	10	10.000

Tests of Equality of Means

	Wilks' Lambda	F	df1	df2	Sig.
INFO	**.269**	**44.086**	**2**	**37**	**.000**
SIMIL	**.626**	**11.047**	**2**	**37**	**.000**
ARITH	**.686**	**8.479**	**2**	**37**	**.001**
VOCAB	**.291**	**45.084**	**2**	**37**	**.000**
COMPRE	**.263**	**51.905**	**2**	**37**	**.000**
DIGSPAN	**.549**	**15.203**	**2**	**37**	**.000**
PICTCOMP	.760	5.858	2	37	.006
PICTARR	**.678**	**8.796**	**2**	**37**	**.001**
BLOCKDES	.841	3.495	2	37	.041
OBJASSEM	.849	3.278	2	37	.049
CODING	.786	5.023	2	37	.012

Note: That all 11 univariate *F* tests are statistically significant at the nominal α=.05. If a is corrected for multiple testing (α = [.05/11] = .0045), only 7 of the tests would be considered statistically significant. That is, PICTCOMP, BLOCKDES, OBJASSEM, and CODING would not be statistically signficant at the corrected α.

First (maximum) Eigenvalue of $\mathbf{HE}^{-1}$ is equal to
Roy's Largest Root (R).
Sum of Eigenvalues equals Hotelling Trace (T).

Wilks' Lambda (W) is functionally related to the Canonical Correlation coefficient $Rc = \sqrt{1-W} = \sqrt{1-.803} = \mathbf{.444}$.
The Pillai's trace (V) is equal to the Squared Canonical Correlations Summed over the J-1 Discriminant Functions necessary to separate the J groups:

$$V = \sum R^2 c = \mathbf{1.138}$$

Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	**15.995**[a]	**98.5**	**98.5**	**.970**
2	**.246**[a]	**1.5**	**100.0**	**.444**

$T = \sum \lambda = \mathbf{16.241}$ $\quad V = \sum R^2 c = \mathbf{1.138}$

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-Square	df	Sig
1 through 2	**.047**	**97.683**	**22**	**.000**
2	**.803**	**7.029**	**10**	**.723**

Standardized Canonical Discriminant Function Coefficients

	Function 1	Function 2
INFO	.778	-.389
SIMIL	.187	-.340
ARITH	.131	.005
VOCAB	.551	.246
COMPRE	.545	-.072
DIGSPAN	.501	.362
PICTCOMP	-.060	.297
PICTARR	.317	-.140
BLOCKDES	-.405	.952
OBJASSEM	.416	-.473
CODING	.258	.193

Standardized Coefficients used to evaluate the Relative Contribution of each variable to the Discriminant Function that separate the groups

Structure Matrix

	Function 1	Function 2
COMPRE	**.419***	-.037
VOCAB	**.390***	.036
INFO	**.386***	-.091
BLOCKDES	.081	.585*
PICTCOMP	.128	.466*
SIMIL	.185	-.442*
DIGSPAN	.222	.362*
ARITH	.164	.341*
CODING	.125	.301*
PICTARR	.170	.214*
OBJASSEM	.104	.114*

Pooled within-group correlations between discriminating variables and standardized canonical discriminant functions

Canonical Discriminant Function Coefficients

	Function 1	Function 2
INFO	.084	-.042
SIMIL	.015	-.027
ARITH	.009	.000
VOCAB	.055	.025
COMPRE	.066	-.009
DIGSPAN	.045	.032
PICTCOMP	-.004	.022
PICTARR	.023	-.010
BLOCKDES	-.033	.077
OBJASSEM	.036	-.041
CODING	.018	.013
(Constant)	-30.996	-4.541

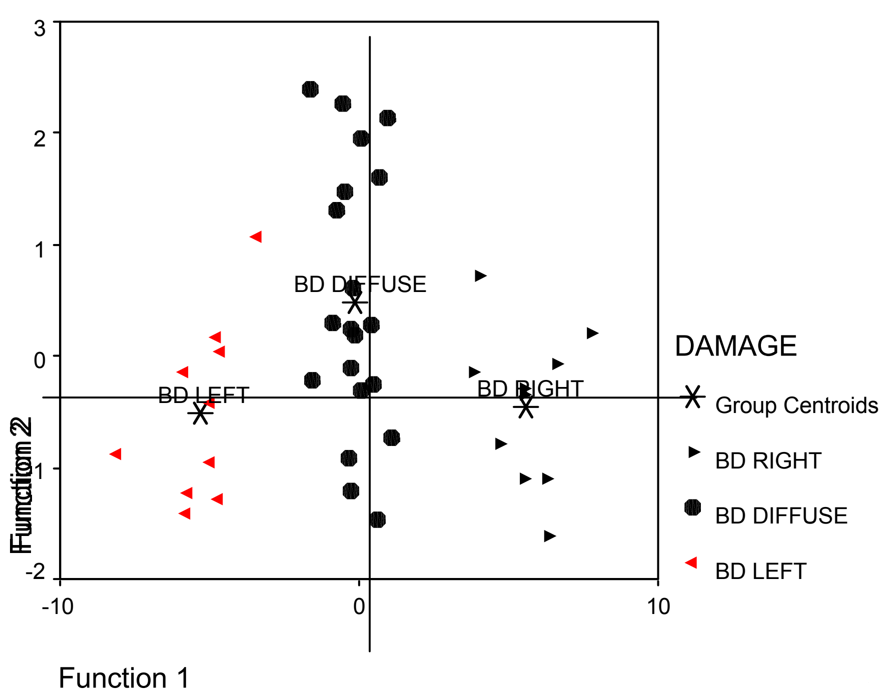
Unstandardized coefficients

$$S = \frac{\sum (Y - \bar{Y}_j)(F - \bar{F}_j)}{\sqrt{\sum (Y - \bar{Y}_j)^2}\sqrt{\sum (F - \bar{F}_j)^2}}$$

Functions at Group Centroids

DAMAGE	Function 1	Function 2
BD LEFT	**-5.282**	**-.503**
BD DIFFUSE	**-.153**	**.476**
BD RIGHT	**5.589**	**-.449**

Canonical Discriminant Functions



Performing Univariate procedures as follow up tests for a significant Omnibus MANOVA

For more than 2 groups, Levin, Serlin, & Seaman (1994) suggested performing post hoc tests (e.g., pairwise comparisons) for only the variables that had significant univariate *F*s (e.g., $p < .05$). One should then correct the level of significance by the number of significant univariate *F*s. In this case all 11 *F*s were statistically significant. For 3 groups there are 3 pairwise comparisons and thus 33 tests will be performed. The Dunn-Sidak correction was applied in the SPSS GLM module. $\alpha_{DS} = 1-(1-\alpha)^{1/33} = 1-(.95)^{1/33} = .001553$. Thus, the Confidence Interval has a width of 1-(.001553/2) = 99.8447%. If the Bonferroni correction was applied the correct alpha would be $\alpha_{BON} = .05/33 = .001515$ and the Confidence Interval would have a width of 99.9242%. Alternately, one could correct the alpha for 11 tests and perform a Tukey HSD, which "automatically" corrects for the number of pairwise comparisons. In any case, inferences are still made at the 95% level of confidence ($p < .05$).

Univariate Simultaneous Test Procedures

LSD

Multiple Comparisons

Dependent Variable	(I) DAMAGE	(J) DAMAGE	Mean Difference (I-J)	Std. Error	Sig.	98.8447% Confidence Interval Lower Bound	98.8447% Confidence Interval Upper Bound
INFO	**BD LEFT**	**BD DIFFUSE**	**-17.500***	**3.5870**	**.000**	**-29.7568**	**-5.2432**
	BD LEFT	**BD RIGHT**	**-38.800***	**4.1419**	**.000**	**-52.9529**	**-24.6471**
	BD DIFFUSE	**BD RIGHT**	**-21.300***	**3.5870**	**.000**	**-33.5568**	**-9.0432**
SIMIL	BD LEFT	BD DIFFUSE	-6.600	4.9423	.190	-23.4878	10.2878
	BD LEFT	**BD RIGHT**	**-25.400***	**5.7069**	**.000**	**-44.9003**	**-5.8997**
	BD DIFFUSE	**BD RIGHT**	**-18.800***	**4.9423**	**.001**	**-35.6878**	**-1.9122**
ARITH	BD LEFT	BD DIFFUSE	-16.450	5.4220	.004	-34.9769	2.0769
	BD LEFT	**BD RIGHT**	**-25.200***	**6.2608**	**.000**	**-46.5930**	**-3.8070**
	BD DIFFUSE	BD RIGHT	-8.750	5.4220	.115	-27.2769	9.7769
VOCAB	**BD LEFT**	**BD DIFFUSE**	**-20.300***	**3.8594**	**.000**	**-33.4875**	**-7.1125**
	BD LEFT	**BD RIGHT**	**-42.300***	**4.4565**	**.000**	**-57.5276**	**-27.0724**
	BD DIFFUSE	**BD RIGHT**	**-22.000***	**3.8594**	**.000**	**-35.1875**	**-8.8125**
COMPRE	**BD LEFT**	**BD DIFFUSE**	**-17.450***	**3.2001**	**.000**	**-28.3845**	**-6.5155**
	BD LEFT	**BD RIGHT**	**-37.600***	**3.6951**	**.000**	**-50.2261**	**-24.9739**
	BD DIFFUSE	**BD RIGHT**	**-20.150***	**3.2001**	**.000**	**-31.0845**	**-9.2155**
DIGSPAN	**BD LEFT**	**BD DIFFUSE**	**-16.750***	**4.3425**	**.001**	**-31.5882**	**-1.9118**
	BD LEFT	**BD RIGHT**	**-27.300***	**5.0143**	**.000**	**-44.4337**	**-10.1663**
	BD DIFFUSE	BD RIGHT	-10.550	4.3425	.020	-25.3882	4.2882
PICTCOM	BD LEFT	BD DIFFUSE	-15.150	5.2649	.007	-33.1401	2.8401
	BD LEFT	BD RIGHT	-19.300	6.0794	.003	-40.0732	1.4732
	BD DIFFUSE	BD RIGHT	-4.150	5.2649	.436	-22.1401	13.8401
PICTARR	BD LEFT	BD DIFFUSE	-15.000	5.3623	.008	-33.3230	3.3230
	BD LEFT	**BD RIGHT**	**-25.800***	**6.1919**	**.000**	**-46.9575**	**-4.6425**
	BD DIFFUSE	BD RIGHT	-10.800	5.3623	.051	-29.1230	7.5230
BLOCKDE	BD LEFT	BD DIFFUSE	-12.150	4.7607	.015	-28.4172	4.1172
	BD LEFT	BD RIGHT	-11.200	5.4972	.049	-29.9838	7.5838
	BD DIFFUSE	BD RIGHT	.950	4.7607	.843	-15.3172	17.2172
OBJASSE	BD LEFT	BD DIFFUSE	-7.550	4.5189	.103	-22.9910	7.8910
	BD LEFT	BD RIGHT	-13.300	5.2180	.015	-31.1297	4.5297
	BD DIFFUSE	BD RIGHT	-5.750	4.5189	.221	-21.1910	9.6910
CODING	BD LEFT	BD DIFFUSE	-13.350	5.5280	.021	-32.2392	5.5392
	BD LEFT	BD RIGHT	-19.600	6.3832	.004	-41.4114	2.2114
	BD DIFFUSE	BD RIGHT	-6.250	5.5280	.265	-25.1392	12.6392