Matrix Algebra Computation of a Correlation Matrix

Suppose N = 5 subjects were measured on p = 3 variables.

The Data Matrix **D** is a matrix of deviation scores with N = 5 rows and p = 3 columns.

$$\mathbf{D}_{(Nxp)} = \begin{bmatrix} 1 & 1.5 & 0 \\ 1 & .5 & 1 \\ 0 & -1 & -1 \\ -1 & -.5 & 1 \\ -1 & -.5 & -1 \end{bmatrix} \qquad \mathbf{D'}_{(pxN)} = \begin{bmatrix} 1 & 1 & 0 & -1 & -1 \\ 1.5 & .5 & -1 & -.5 & -.5 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix}$$

$\mathbf{D'}_{(KxN)}\mathbf{D}_{(NxK)} = \mathbf{V}_{(KxK)}$ = Variance-Covariance matrix. The correlation matrix $\mathbf{R}_{(pxp)} = \dfrac{\mathbf{V}_{(pxp)}}{(N-1)}$

$$\mathbf{D'}_{(pxN)} \begin{bmatrix} 1 & 1 & 0 & -1 & -1 \\ 1.5 & .5 & -1 & -.5 & -.5 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \times \mathbf{D}_{(Nxp)} \begin{bmatrix} 1 & 1.5 & 0 \\ 1 & .5 & 1 \\ 0 & -1 & -1 \\ -1 & -.5 & 1 \\ -1 & -.5 & -1 \end{bmatrix} =$$

r1 c1 (r1x c1)

r1	c1	
1	1	1
1	1	1
0	0	0
-1	-1	1
-1	-1	1
		4

r1 c2 (r1x c2)

r1	c2	
1	1.5	1.5
1	.5	.5
0	-1	0
-1	-.5	.5
-1	-.5	.5
		3

$\mathbf{V}_{(pxp)}$

Σ(r1xc1) (1x1) +(1x1) +(0x0) +(-1x-1) +(-1x-1)	Σ(r1xc2) (1x1.5) +(1x.5) +(0x-1) +(-1x-.5) +(-1x-.5)	Σ(r1xc3) (1x0) +(1x1) +(0x-1) +(-1x1) +(-1x-1)
= 4	**= 3**	**= 1**
Σ(r2xc1) (1.5x1) +(.5x1) +(-1x0) +(-.5x-1) +(-.5x-1)	Σ(r2xc2) (1.5x1.5) +(.5x.5) +(-1x-1) +(-.5x-.5) +(-.5x-.5)	Σ(r2xc3) (1.5x0) +(.5x1) +(-1x-1) +(-.5x1) +(-.5x-1)
= 3	**= 4**	**= 1.5**
Σ(r3xc1) (0x1) +(1x1) +(-1x0) +(1x-1) +(-1x-1)	Σ(r3xc2) (0x1.5) +(1x.5) +(-1x-1) +(1x-.5) +(-1x-.5)	Σ(r3xc3) (0x0) +(1x1) +(-1x-1) +(1x1) +(-1x-1)
= 1	**= 1.5**	**= 4**

$$\mathbf{R}_{(pxp)} = \begin{bmatrix} 1.000 & 0.750 & 0.250 \\ 0.750 & 1.000 & 0.375 \\ 0.250 & 0.375 & 1.000 \end{bmatrix}$$

```
data one;
input y x1 x2 x3;
zy=(y-5.66667)/1.0380;
z1=(x1-5)/1.41421;
z2=(x2-6.5)/1.87083;
z3=(x3-3)/1.4121;
cards;
7 5 9 5
6 7 7 3
6 6 8 3
6 5 5 4
5 4 6 1
4 3 4 2
proc corr; var y x1 x2 x3;run;
proc reg;model y = x1 x2 x3 / i xpx stb tol vif pcorr1 scorr1 pcorr2 scorr2 influence; run;
```

The CORR Procedure

4 Variables: y x1 x2 x3

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
y	6	5.66667	1.03280	34.00000	4.00000	7.00000
x1	6	5.00000	1.41421	30.00000	3.00000	7.00000
x2	6	6.50000	1.87083	39.00000	4.00000	9.00000
x3	6	3.00000	1.41421	18.00000	1.00000	5.00000

Pearson Correlation Coefficients, N = 6
Prob > |r| under H0: Rho=0

	y	x1	x2	x3
y	**1.00000**	**0.68465** 0.1335	**0.82808** 0.0418	**0.82158** 0.0449
x1	**0.68465** 0.1335	**1.00000**	**0.60474** 0.2035	**0.40000** 0.4320
x2	**0.82808** 0.0418	**0.60474** 0.2035	**1.00000**	**0.52915** 0.2804
x3	**0.82158** 0.0449	**0.40000** 0.4320	**0.52915** 0.2804	**1.00000**

r_{YX} vector

R_X submatix

The REG Procedure
Model: MODEL1

Model Crossproducts X'X X'Y Y'Y

Variable	Intercept	x1	x2	x3	y
Intercept	6	30	39	18	34
x1	30	160	203	94	175
x2	39	203	271	124	229
x3	18	94	124	64	108
y	34	175	229	108	198

X'X → (X'X block); X'Y → (y column); Y'Y → 198

Model: MODEL1
Dependent Variable: y

X'X Inverse, Parameter Estimates, and SSE

Variable	Intercept	x1	x2	x3	y
Intercept	3.2326565144	-0.317258883	-0.218274112	-0.020304569	2.2123519459
x1	-0.317258883	0.1598984772	-0.065989848	-0.017766497	0.1649746193
x2	-0.218274112	-0.065989848	0.1065989848	-0.04822335	0.2335025381
x3	-0.020304569	-0.017766497	-0.04822335	0.1408629442	0.3705583756
y	2.2123519459	0.1649746193	0.2335025381	0.3705583756	**0.4170896785**

$(X'X)^{-1}$ → (inverse block); $\hat{\beta}$ → (y column); SSE → **0.4170896785**

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	4.91624	1.63875	7.86	0.1150
Error	2	**0.41709**	0.20854		
Corrected Total	5	5.33333			

SSE → **0.41709**

Root MSE	0.45667	R-Square	**0.9218**
Dependent Mean	5.66667	Adj R-Sq	0.8045
Coeff Var	8.05883		

$$R^2 = \frac{SS_{Model}}{SS_{Total}} = 4.91624/5.33333 = .9218$$

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits		Standardized Estimate
Intercept	1	2.21235	0.82107	2.69	0.1145	-1.32042	5.74513	0
x1	1	0.16497	0.18261	0.90	0.4617	-0.62073	0.95068	0.22590
x2	1	0.23350	0.14910	1.57	0.2578	-0.40802	0.87503	0.42297
x3	1	0.37056	0.17139	2.16	0.1631	-0.36689	1.10801	0.50741

Variable	Squared Semi-partial Corr Type I	Squared Partial Corr Type I	Squared Semi-partial Corr Type II	Squared Partial Corr Type II	Tolerance	Variance Inflation
Intercept	.	.	.	.	.	0
x1	0.46875	0.46875	0.03191	0.28982	0.62540	1.59898
x2	0.27027	0.50874	0.09590	0.55083	0.53605	1.86548
x3	0.18278	0.70034	0.18278	0.70034	0.70991	1.40863

Obs	Dependent Variable	Predicted Value	Residual	RStudent	Hat Diag H	DFFITS
1	7.0000	6.9915	0.008460	0.0447	0.9141	0.1460
2	6.0000	6.1134	-0.1134	-0.3389	0.7009	-0.5189
3	6.0000	6.1819	-0.1819	-0.3790	0.3684	-0.2895
4	6.0000	5.6870	0.3130	1.7937	0.6920	2.6889
5	5.0000	4.6438	0.3562	4.8982	0.6832	7.1925
6	4.0000	4.3824	-0.3824	-6.5735	0.6413	-8.7892

DFFITS = Rstudent $\sqrt{h_{ii}/(1-h_{ii})}$ DFFITS(1) = (0.0447) $\sqrt{0.9141/(1-0.9141)}$ = 0.1460

Obs	Student Residual	Cook's D	DFBETAS Intercept	x1	x2	x3
1	0.0632	0.011	-0.0356	-0.0766	0.0795	0.0656
2	-0.454	0.121	0.1989	-0.4445	0.1493	0.0985
3	-0.501	0.037	0.1268	-0.0726	-0.1372	0.1145
4	1.235	0.857	0.8517	0.6565	-2.0603	1.8360
5	1.386	1.035	3.0669	-1.9884	2.9088	-5.5610
6	-1.398	0.874	-8.3458	3.7618	2.9009	-0.4453

SAS Reports scaled (standardized) DFBETAS. These are NOT the actual change in the coefficients by removing the i^{th} case, rather if one these |DFBETAs| is greater than 2 then may be considered to have a "significant" influence on the regression coefficient.

Cook's D = (Student Residual)^2 h_{ii} /(1-h_{ii})(p+1).
Cook's D(1) = (0.0632)^2(0.91411)/(1-0.9141)(4) = 001064

Sum of Residuals	0
Sum of Squared Residuals	0.41709
Predicted Residual SS (PRESS)	3.66977

K = number of regression terms (intercepts and slopes) = number of predictor variables plus one (p +1).

$$\mathbf{Y} = \mathbf{X}\hat{\beta} + \mathbf{E}$$

$$\underset{(N\times1)}{\mathbf{Y}} = \begin{bmatrix} 7 \\ 6 \\ 6 \\ 6 \\ 5 \\ 4 \end{bmatrix} \quad \underset{(N\times K)}{\mathbf{X}} = \begin{bmatrix} 1 & 5 & 9 & 5 \\ 1 & 7 & 7 & 3 \\ 1 & 6 & 8 & 3 \\ 1 & 5 & 5 & 4 \\ 1 & 4 & 6 & 1 \\ 1 & 3 & 4 & 2 \end{bmatrix} \quad \mathbf{X'Y} = \underset{(K\times N)}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 7 & 6 & 5 & 4 & 3 \\ 9 & 7 & 8 & 5 & 6 & 4 \\ 5 & 3 & 3 & 4 & 1 & 2 \end{bmatrix}} \underset{(N\times1)}{\begin{bmatrix} 7 \\ 6 \\ 6 \\ 6 \\ 5 \\ 4 \end{bmatrix}} = \underset{(K\times1)}{\begin{bmatrix} 34 \\ 175 \\ 229 \\ 108 \end{bmatrix}}$$

$$\mathbf{X'X} = \underset{(K\times N)}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 7 & 6 & 5 & 4 & 3 \\ 9 & 7 & 8 & 5 & 6 & 4 \\ 5 & 3 & 3 & 4 & 1 & 2 \end{bmatrix}} \underset{(N\times K)}{\begin{bmatrix} 1 & 5 & 9 & 5 \\ 1 & 7 & 7 & 3 \\ 1 & 6 & 8 & 3 \\ 1 & 5 & 5 & 4 \\ 1 & 4 & 6 & 1 \\ 1 & 3 & 4 & 2 \end{bmatrix}} = \underset{(K\times K)}{\begin{bmatrix} 6 & 30 & 39 & 18 \\ 30 & 160 & 203 & 94 \\ 39 & 203 & 271 & 124 \\ 18 & 94 & 124 & 64 \end{bmatrix}}$$

$$(\mathbf{X'X})^{-1} = \underset{(K\times K)}{\begin{bmatrix} 3.2327 & -0.3173 & -0.2183 & -0.0203 \\ -0.3173 & 0.1599 & -0.0660 & -0.0178 \\ -0.2183 & -0.0660 & 0.1066 & -0.0482 \\ -0.0203 & -0.0178 & -0.0482 & 0.1409 \end{bmatrix}}$$

$$\hat{\beta} = [(\mathbf{X'X})^{-1}(\mathbf{X'Y})] = \underset{(K\times K)}{\begin{bmatrix} 3.2327 & -0.3173 & -0.2183 & -0.0203 \\ -0.3173 & 0.1599 & -0.0660 & -0.0178 \\ -0.2183 & -0.0660 & 0.1066 & -0.0482 \\ -0.0203 & -0.0178 & -0.0482 & 0.1409 \end{bmatrix}} \underset{(K\times1)}{\begin{bmatrix} 34 \\ 175 \\ 229 \\ 108 \end{bmatrix}} = \underset{(K\times1)}{\begin{bmatrix} 2.2124 \\ 0.1650 \\ 0.2335 \\ 0.3706 \end{bmatrix}}$$

$$\underset{(N\times1)}{\hat{\mathbf{Y}}} = \mathbf{X}\hat{\beta} = \underset{(N\times K)}{\begin{bmatrix} 1 & 5 & 9 & 5 \\ 1 & 7 & 7 & 3 \\ 1 & 6 & 8 & 3 \\ 1 & 5 & 5 & 4 \\ 1 & 4 & 6 & 1 \\ 1 & 3 & 4 & 2 \end{bmatrix}} \underset{(K\times1)}{\begin{bmatrix} 2.2124 \\ 0.1650 \\ 0.2335 \\ 0.3706 \end{bmatrix}} = \underset{(N\times1)}{\begin{bmatrix} 6.9915 \\ 6.1134 \\ 6.1819 \\ 5.6870 \\ 4.6438 \\ 4.3824 \end{bmatrix}}$$

$$\underset{(N\times1)}{\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}}} = \begin{bmatrix} 7 \\ 6 \\ 6 \\ 6 \\ 5 \\ 4 \end{bmatrix} - \underset{(N\times1)}{\begin{bmatrix} 6.9915 \\ 6.1134 \\ 6.1819 \\ 5.6870 \\ 4.6438 \\ 4.3824 \end{bmatrix}} = \underset{(N\times1)}{\begin{bmatrix} 0.0084 \\ -0.1134 \\ -0.1819 \\ 0.3130 \\ 0.3562 \\ -0.3824 \end{bmatrix}}$$

$$\underset{(1\times1)}{\mathbf{SSE} = \mathbf{e'e}} = \underset{(N\times1)}{\begin{bmatrix} 0.0084 & -0.1134 & -0.1819 & 0.3130 & 0.3562 & -0.3824 \end{bmatrix}} \underset{(N\times1)}{\begin{bmatrix} 0.0084 \\ -0.1134 \\ -0.1819 \\ 0.3130 \\ 0.3562 \\ -0.3824 \end{bmatrix}} \boxed{= \mathbf{0.4171}}$$

$E(\hat{\beta}) = E[(\mathbf{X'X})^{-1}(\mathbf{X'Y})]$). Define $\mathbf{T} = [(\mathbf{X'X})^{-1}\mathbf{X'}]$, so $E(\hat{\beta}) = \mathbf{T}E(\mathbf{Y})$. With $E(\mathbf{Y}) = \mathbf{X\beta}$

$E(\hat{\beta}) = \mathbf{TX\beta}$; thus substituting for $\mathbf{T}$, $E(\hat{\beta}) = (\mathbf{X'X})^{-1}(\mathbf{X'X})\,\boldsymbol{\beta} = \mathbf{I\beta}$ and $E(\hat{\beta}) = \boldsymbol{\beta}$

$COV(\hat{\beta}) = [\hat{\beta} - E(\hat{\beta})][\hat{\beta} - E(\hat{\beta})]'\,/df$; Define $[\hat{\beta} - E(\hat{\beta})] = \mathbf{T}[\mathbf{Y} - E(\mathbf{Y})] = \mathbf{T}[\mathbf{Y} - \mathbf{X\beta}] = \mathbf{Te}$

$COV(\hat{\beta}) = \mathbf{Tee'T'}\,/df$. With $\mathbf{ee'}/df = COV(\mathbf{e}) = s_e^2\,\mathbf{I}_N$, by definition of Independence.

$COV(\hat{\beta}) = \mathbf{T}\,s_e^2\,\mathbf{I}_N\mathbf{T'} = \mathbf{T}\,s_e^2\,\mathbf{T'}$ with s_e^2 being a scalar, $COV(\hat{\beta}) = s_e^2\,\mathbf{TT'}$

Substituting for $\mathbf{T}$, $COV(\hat{\beta}) = s_e^2\,(\mathbf{X'X})^{-1}\underline{\mathbf{X'X}}(\mathbf{X'X})^{-1} = s_e^2\,(\mathbf{X'X})^{-1}\mathbf{I} = s_e^2\,(\mathbf{X'X})^{-1}$

$COV(\hat{\beta})=$
$(\mathbf{SSE}/([N-K])(\mathbf{X'X})^{-1} = \frac{.4171}{(6-4)}$
$\mathbf{MSE}(\mathbf{X'X})^{-1} =$

3.2327	-0.3173	-0.2183	-0.0203
-0.3173	0.1599	-0.0660	-0.0178
-0.2183	-0.0660	0.1066	-0.0482
-0.0203	-0.0178	-0.0482	0.1409

(KxK)

$COV(\hat{\beta})=$

0.6742	-0.0662	-0.0455	-0.0042
-0.0662	**0.0333**	-0.0138	-0.0037
-0.0455	-0.0138	**0.0222**	-0.0101
-0.0042	-0.0037	-0.0101	**0.0294**

(KxK)

Diagonal Elements are the Variance of the Regression Coefficients.

$$t(\hat{\beta}_2) = \frac{\hat{\beta}_2}{\sqrt{\mathbf{VAR}(\hat{\beta}_2)}} = \frac{.2335}{\sqrt{0.0222}} = 1.566$$

Standardized Solution: $\mathbf{R_X}$ **= correlation matrix for the X variables**
$\mathbf{r_{yx}}$ **= vector of correlations between Y and each X**

$$\mathbf{R_X} = \begin{bmatrix} 1 & 0.60474 & 0.40000 \\ 0.60474 & 1 & 0.52915 \\ 0.40000 & 0.52915 & 1 \end{bmatrix}_{(pxp)} \qquad \mathbf{r_{yx}} = \begin{bmatrix} 0.68465 \\ 0.82808 \\ 0.82158 \end{bmatrix}_{(px1)}$$

$$\hat{\beta} = [(\mathbf{R_X}^{-1})(\mathbf{r_{yx}})] = \begin{bmatrix} \mathbf{1.5990} & -0.8730 & -0.1777 \\ -0.8730 & \mathbf{1.8655} & -0.6379 \\ -0.1777 & -0.6379 & \mathbf{1.4086} \end{bmatrix}_{(pxp)} \begin{bmatrix} 0.68465 \\ 0.82808 \\ 0.82158 \end{bmatrix}_{(px1)} = \begin{bmatrix} \mathbf{0.22590} \\ \mathbf{0.42297} \\ \mathbf{0.50741} \end{bmatrix}_{(px1)}$$

Diagonal elements are Variance Inflation Factors

$$\mathbf{R^2} = (\mathbf{r_{yx}})'\hat{\beta} = \begin{bmatrix} .22590 & .42297 & .50741 \end{bmatrix}_{(1xp)} \begin{bmatrix} 0.68465 \\ 0.82808 \\ 0.82158 \end{bmatrix}_{(px1)} = \mathbf{0.92179}_{(1x1)}$$

With $\text{COV}(\hat{\beta}) = s_e^2\ (\mathbf{X'X})^{-1}$ and all Variances = 1 then $s_e^2 = (1 - R^2)/dfe$

Then, $\text{COV}(\hat{\beta}) = [(1 - R^2)/dfe]\mathbf{R_X}^{-1}$ and the variance is $\text{VAR}(\hat{\beta}_j) = (1 - R^2)(\text{VIF}_j)/dfe$

$$\mathbf{VAR}(\hat{\beta}_j) = \frac{(1- R^2)(\text{VIF}_j)}{(N-p-1)} = \frac{\{1-[(\mathbf{r_{yx}})'(\mathbf{R_X}^{-1})(\mathbf{r_{yx}})]\}\mathbf{R_X}^{-1}{}_{jj}}{(N-p-1)}$$

$$\mathbf{VAR}(\hat{\beta}_2) = \frac{(.07821)(1.8655)}{(6-3-1)} = .07295 \qquad t(\hat{\beta}_2) = \frac{\hat{\beta}_2}{\sqrt{\mathbf{VAR}(\hat{\beta}_2)}} = \frac{.42297}{.27009} = 1.566$$

HAT Matrix

$$\underset{(N\times N)}{\mathbf{H}} = [\mathbf{X}(\mathbf{X'X})^{-1}\mathbf{X'}] = \underset{(N\times K)}{\begin{bmatrix} 1 & 5 & 9 & 5 \\ 1 & 7 & 7 & 3 \\ 1 & 6 & 8 & 3 \\ 1 & 5 & 5 & 4 \\ 1 & 4 & 6 & 1 \\ 1 & 3 & 4 & 2 \end{bmatrix}} \underset{(K\times K)}{\begin{bmatrix} 3.2327 & -0.3173 & -0.2183 & -0.0203 \\ -0.3173 & 0.1599 & -0.0660 & -0.0178 \\ -0.2183 & -0.0660 & 0.1066 & -0.0482 \\ -0.0203 & -0.0178 & -0.0482 & 0.1409 \end{bmatrix}} \underset{(K\times N)}{\begin{bmatrix} 1 & 5 & 9 & 5 & 1 & 5 \\ 1 & 7 & 7 & 3 & 1 & 7 \\ 1 & 6 & 8 & 3 & 1 & 6 \\ 1 & 5 & 5 & 4 & 1 & 5 \end{bmatrix}}$$

$$\underset{(N\times N)}{\mathbf{H}} = \begin{bmatrix} \mathbf{0.9141} & -0.1493 & 0.2212 & 0.0728 & -0.0402 & -0.0186 \\ -0.1493 & \mathbf{0.7009} & 0.3354 & 0.2250 & 0.0385 & -0.1506 \\ 0.2212 & 0.3354 & \mathbf{0.3684} & -0.0643 & 0.2390 & -0.0998 \\ 0.0728 & 0.2250 & -0.0643 & \mathbf{0.6920} & -0.2369 & 0.3113 \\ -0.0402 & 0.0385 & 0.2390 & -0.2369 & \mathbf{0.6832} & 0.3164 \\ -0.0186 & -0.1506 & -0.0998 & 0.3113 & 0.3164 & \mathbf{0.6413} \end{bmatrix}$$

$$\text{DFBETA}_{(i)} = \underset{(K\times K)}{(\mathbf{X'X})^{-1}} \; \underset{(K\times 1)}{\mathbf{X'}_i} \; \underset{(1\times 1)}{(\mathbf{E}_i/(1-\mathbf{H}_{ii}))}$$

$$[(\mathbf{X'X})^{-1}\,\mathbf{X'}_{(1)} = \underset{(K\times K)}{\begin{bmatrix} 3.2327 & -0.3173 & -0.2183 & -0.0203 \\ -0.3173 & 0.1599 & -0.0660 & -0.0178 \\ -0.2183 & -0.0660 & 0.1066 & -0.0482 \\ -0.0203 & -0.0178 & -0.0482 & 0.1409 \end{bmatrix}} \underset{(K\times 1)}{\begin{bmatrix} 1 \\ 5 \\ 9 \\ 5 \end{bmatrix}} = \underset{(K\times 1)}{\begin{bmatrix} -0.4196 \\ -0.2005 \\ 0.17005 \\ 0.16117 \end{bmatrix}}$$

$$\text{DFBETA}_{(1)} = [(\mathbf{X'X})^{-1}\,\mathbf{X'}_{(1)}\,(\mathbf{e}_1/(1-\mathbf{H}_{11}))] = \underset{(K\times 1)}{\begin{bmatrix} -0.4196 \\ -0.2005 \\ 0.17005 \\ 0.16117 \end{bmatrix}} \underset{(1\times 1)}{\frac{0.00846}{(1-.9141)}} = \underset{(K\times 1)}{\begin{bmatrix} \mathbf{-0.0413} \\ \mathbf{-0.0198} \\ \mathbf{0.0168} \\ \mathbf{0.0159} \end{bmatrix}}$$

$$\text{DFBETA}^{*}_{j(i)} = \text{DFBETA}_{j(i)} / [(\mathbf{X'X})^{-1}_{jj}]^{1/2} [(\mathbf{SSE}-[\mathbf{E}^2_i/(1-\mathbf{H}_{ii})])/(N-K-1)]^{1/2}$$

$$(\mathbf{X'X})^{-1}_{jj} = \underset{(K\times 1)}{\begin{bmatrix} 3.2327 \\ 0.1599 \\ 0.1066 \\ 0.1409 \end{bmatrix}} \qquad [(\mathbf{X'X})^{-1}_{jj}]^{1/2} = \underset{(K\times 1)}{\begin{bmatrix} 1.7980 \\ 0.3999 \\ 0.3265 \\ 0.3754 \end{bmatrix}}$$

$$\text{RMSE}_{(i)} = [(\mathbf{SSE}-[\mathbf{e}^2_1/(1-\mathbf{H}_{11})])/(N-K-1)]^{1/2} = \left[\frac{.4171 - \dfrac{(0.00846)^2}{(1-.9141)}}{(6-4-1)}\right]^{1/2} = (\mathbf{.4163})^{1/2} = \mathbf{.6452}$$

$$[(\mathbf{X'X})^{-1}_{jj}]^{1/2}(\mathbf{.6452}) = \underset{(K\times 1)}{\begin{bmatrix} 1.1601 \\ 0.2580 \\ 0.2107 \\ 0.2422 \end{bmatrix}}$$

$$\text{DFBETA}^{*}_{(1)} = [(\mathbf{X'X})^{-1}\,\mathbf{X'}_{(1)}\,(\mathbf{e}_1/(1-\mathbf{H}_{11}))] = \underset{(K\times 1)}{\begin{bmatrix} -0.0413 \\ -0.0198 \\ 0.0168 \\ 0.0159 \end{bmatrix}} \div \underset{(K\times 1)}{\begin{bmatrix} 1.1601 \\ 0.2580 \\ 0.2107 \\ 0.2422 \end{bmatrix}} = \underset{(K\times 1)}{\begin{bmatrix} \mathbf{-0.0356} \\ \mathbf{-0.0766} \\ \mathbf{0.0795} \\ \mathbf{0.0656} \end{bmatrix}}$$

Regression ANALYSIS with subject 1 omitted

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	2.78374	0.92791	2.23	0.4491
Error	1	0.41626	**0.41626**		
Corrected Total	4	3.20000			

Root MSE	**0.64518**	R-Square	0.8699
Dependent Mean	5.40000	Adj R-Sq	0.4797
Coeff Var	11.94776		

$RMSE_{(i)}$ Computed on Page 8

Parameter Estimates without Subject 1

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Standardized Estimate
Intercept	1	2.25369	1.48297	1.52	0.3705	0
x1	1	0.18473	0.51131	0.36	0.7793	0.32656
x2	1	0.21675	0.42959	0.50	0.7025	0.38316
x3	1	0.35468	0.42959	0.83	0.5606	0.45213

Parameter Estimates with Subject 1 (reproduced from Page 4)

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Standardized Estimate
Intercept	1	2.21235	0.82107	2.69	0.1145	0
x1	1	0.16497	0.18261	0.90	0.4617	0.22590
x2	1	0.23350	0.14910	1.57	0.2578	0.42297
x3	1	0.37056	0.17139	2.16	0.1631	0.50741

Difference in Parameter Estimates

Variable	DF	(**With subject 1**) Parameter Estimate	(**Without subject 1**) Parameter Estimate	**Difference** DFBETA
Intercept	1	2.21235	2.25369	**-0.04134**
x1	1	0.16497	0.18473	**-0.01976**
x2	1	0.23350	0.21675	**0.01675**
x3	1	0.37056	0.35468	**0.01588**

$DFBETA_{(1)}$ Computed on Page 8.

Cook's D can be derived as a function of the DFBETAs, thus is a summary measure of how overall model fit is affected by the i^{th} case.

Cook's $D_{(i)} = (\mathbf{DFBETA}_{(i)})'\mathbf{X}_{(i)}'\mathbf{X}_{(i)}(\mathbf{DFBETA}_{(i)})/(p+1)(MSE)(h_{ii})$

Cook's $D_{(1)} = \dfrac{[-0.4134 - 0.1976(5) + 0.01675(9) + 0.01588(5)]^2}{(4)(0.20854)(0.9141)} = \dfrac{0.0081018}{0.7625056} = 0.01064$

```
data regmat;
proc iml;
y={7,6,6,6,5,4};
ym=y[:,];z=y-ym;
sdy=((z`*z)/(nrow(y)-1))##.5;
z=z/sdy;
xd={5 9 5,
    7 7 3,
    6 8 3,
    5 5 4,
    4 6 1,
    3 4 2};
n=nrow(xd);
v=ncol(xd);
k=ncol(xd)+1;
dfe=n-k;
xdm=xd[:,];print xdm;
xdm=(j(6,1,1))*xdm;
zx=xd-xdm;print zx;
vardx=((zx`*zx)/(n-1));
sdx=(vecdiag(vardx))##.5;
sdx=(j(6,1,1))*sdx`;
zx=zx/sdx;
Rx=zx`*zx/((n-1));
Ry=(zx`*z)/(n-1);print Ry;
Rxinv=inv(Rx);
Beta=Rxinv*Ry;
Rsq=Beta`*ry;
zyx=z||zx;
Ryx=(zyx`*zyx)/(n-1);
Print Ryx;
Sebeta=(((vecdiag(Rxinv))#(1-Rsq))/dfe)##.5;
  x=(j(6,1,1))||(xd);print x;
s=X`*x;sinv=inv(s);print s sinv;
xy=X`*Y;
print xy;
b=Sinv*xy;
HAT=x*Sinv*x`;
print HAT;
tracehat=trace(HAT);
hii=vecdiag(HAT);
yhat=X*B;
print yhat;
resid=y-yhat;
sse=resid`*resid;
rstud=resid/((mse##.5)*(sqrt(1-hii)));
cookd=((rstud##2)#(hii/(1-hii)))/4;
print resid rstud hii cookd sse;
mse=sse/dfe;
covb=sinv#mse;
seb=(vecdiag(covb))##.5;
tb=b/seb;
tbeta=beta/sebeta;
pb=2*(1-probt(tb,dfe));
pbeta=2*(1-probt(tbeta,dfe));
print mse covb;
print Rsq;
F=(Rsq/v)/((1-Rsq)/(dfe));
pF=1-probf(F,v,dfe);
print F pF;
print b seb tb pb;
print beta sebeta tbeta pbeta;

do i = 1 to n;
dfbetai=sinv*(x[i,])`*(resid[i,]/(1-HAT[i,i]));
dfbetas=(dfbetai/((vecdiag(sinv))##.5))
   /(((sse-((resid[i,]##2)/(1-HAT[i,i])))/(n-k-1))##.5);
cooks=(dfbetai`*(x[i,])`*(x[i,])*dfbetai)/(mse*k*(HAT[i,i]));
if i = 1 then dvec=i||cooks||(dfbetai`)||(dfbetas`);
else dvec=dvec//(i||cooks||(dfbetai`)||(dfbetas`));
end;
print dvec;
```

```
data one;
input y x1 x2 x3;
cards;
7 5 9 5
6 7 7 3
6 6 8 3
6 5 5 4
5 4 6 1
4 3 4 2
;
proc standard data=one mean=0 std=1 out=stand;
    var y x1 x2 x3;
run;
data stand;set stand;
zy=y;z1=x1;z2=x2;z3=x3;
keep zy z1 z2 z3;
run;
proc print data=stand;
proc reg data=one;
model y = x1 x2 x3 / stb;
output out =one predicted=yhat residual=ey;run;
proc reg data=stand;
model zy = z1 z2 z3 / stb;
output out =stand predicted=zhat residual=ez;run;
proc means data=one n mean std var; var y yhat ey;run;
proc means data=stand n mean std var; var zy zhat ez;run;
proc corr data=one cov pearson;var y yhat ey x1 x2 x3;run;
proc corr data=stand pearson;var zy zhat ez z1 z2 z3;run;
```

Obs	zy	z1	z2	z3
1	1.29099	0.00000	1.33631	1.41421
2	0.32275	1.41421	0.26726	0.00000
3	0.32275	0.70711	0.80178	0.00000
4	0.32275	0.00000	-0.80178	0.70711
5	-0.64550	-0.70711	-0.26726	-1.41421
6	-1.61374	-1.41421	-1.33631	-0.70711

```
proc reg data=one; model y = x1 x2 x3 / stb;output out =one predicted=yhat residual=ey;run;
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	4.91624	1.63875	7.86	0.1150
Error	2	0.41709	0.20854		
Corrected Total	5	5.33333			

Root MSE	0.45667	R-Square	0.9218
Dependent Mean	5.66667	Adj R-Sq	0.8045

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Standardized Estimate
Intercept	1	2.21235	0.82107	2.69	0.1145	0
x1	1	0.16497	0.18261	0.90	0.4617	0.22590
x2	1	0.23350	0.14910	1.57	0.2578	0.42297
x3	1	0.37056	0.17139	2.16	0.1631	0.50741

```
proc reg data=stand;model zy = z1 z2 z3 / stb;output out =stand predicted=zhat residual=ez;run;
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	4.60898	1.53633	7.86	0.1150
Error	2	0.39102	0.19551		
Corrected Total	5	5.00000			

Root MSE	0.44217	R-Square	0.9218
Dependent Mean	-2.5905E-16	Adj R-Sq	0.8045

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Standardized Estimate
Intercept	1	-2.5905E-16	0.18051	-0.00	1.0000	0
z1	1	0.22590	0.25005	0.90	0.4617	0.22590
z2	1	0.42297	0.27008	1.57	0.2578	0.42297
z3	1	0.50741	0.23469	2.16	0.1631	0.50741

The MEANS Procedure

Variable	N	Mean	Std Dev	Variance
yhat	6	5.6666667	0.9915890	0.9832487
ey	6	-2.77556E-16	0.2888216	0.0834179
y	6	5.6666667	1.0327956	1.0666667

Covariance(Y,Ŷ) = Variance(Ŷ)

$$\frac{\text{Var(Yhat)}}{\text{Var(Y)}} = \frac{0.9832487}{1.0666667} = 0.9218 = R^2$$

Covariance Matrix, DF = 5

	y	yhat	ey
y	1.066666667	0.983248731	0.083417936
yhat	0.983248731	0.983248731	0.000000000
ey	0.083417936	0.000000000	0.083417936

Covariance(Y,Ŷ) = Variance(Ŷ)

Covariance(Y,e) = Variance(e)

$Corr(Y,\hat{Y})^2 = (0.96010)^2 = 0.9218$

Residual uncorrelated (orthogonal) to Ŷ and each X that composes Ŷ

	y	yhat	ey	x1	x2	x3
y	1.00000	0.96010	0.27965	0.68465	0.82808	0.82158
		0.0024	0.5915	0.1335	0.0418	0.0449
yhat	0.96010	1.00000	0.00000	0.71310	0.86249	0.85573
	0.0024		1.0000	0.1117	0.0271	0.0297
ey	0.27965	0.00000	1.00000	0.00000	0.00000	0.00000
	0.5915	1.0000		1.0000	1.0000	1.0000
x1	0.68465	0.71310	0.00000	1.00000	0.60474	0.40000
	0.1335	0.1117	1.0000		0.2035	0.4320
x2	0.82808	0.86249	0.00000	0.60474	1.00000	0.52915
	0.0418	0.0271	1.0000	0.2035		0.2804
x3	0.82158	0.85573	0.00000	0.40000	0.52915	1.00000
	0.0449	0.0297	1.0000	0.4320	0.2804	

When Standardized

Variable	N	Mean	Std Dev	Variance	
zy	6	-2.59052E-16	1.0000000	1.0000000	
zhat	6	-2.59052E-16	0.9601019 **SD(Ž) =$R_{zž}$**	0.9217957	**= R^2 = Covariance(Z,Ž) = Variance(Ž)**
ez	6	-1.85037E-17	0.2796503 **SD(e) =R_{ze}**	0.0782043	**= (1 − R^2) = Cov(Z,e) = Var(e)**

Covariance Matrix, DF = 5

Covariance(Z,Ž) = Variance(Ž) = R^2

	zy	zhat	ez	
zy	1.000000000	0.921795685	0.078204315	**= Covariance(Z,e) = Variance(e)**
zhat	0.921795685	0.921795685	0.000000000	
ez	0.078204315	0.000000000	0.078204315	

Corr(Z,Ž)^2 = (0.96010)^2
= 0.9218
Corr(Z,Ž) = 0.9601 = SD(Ž)

Residual uncorrelated (orthogonal) to Ŷ and each X that composes Ŷ

SD(e) =R_{ze}

	zy	zhat	**ez**	z1	z2	z3
zy	1.00000 0.0024	0.96010 0.5915	**0.27965** 0.1335	0.68465 0.0418	0.82808 0.0449	0.82158
zhat	0.96010 0.0024	1.00000	0.00000 1.0000	0.71310 0.1117	0.86249 0.0271	0.85573 0.0297
ez	0.27965 0.5915	0.00000 1.0000	1.00000	0.00000 1.0000	0.00000 1.0000	0.00000 1.0000
z1	0.68465 0.1335	0.71310 0.1117	0.00000 1.0000	1.00000	0.60474 0.2035	0.40000 0.4320
z2	0.82808 0.0418	0.86249 0.0271	0.00000 1.0000	0.60474 0.2035	1.00000	0.52915 0.2804
z3	0.82158 0.0449	0.85573 0.0297	0.00000 1.0000	0.40000 0.4320	0.52915 0.2804	1.00000