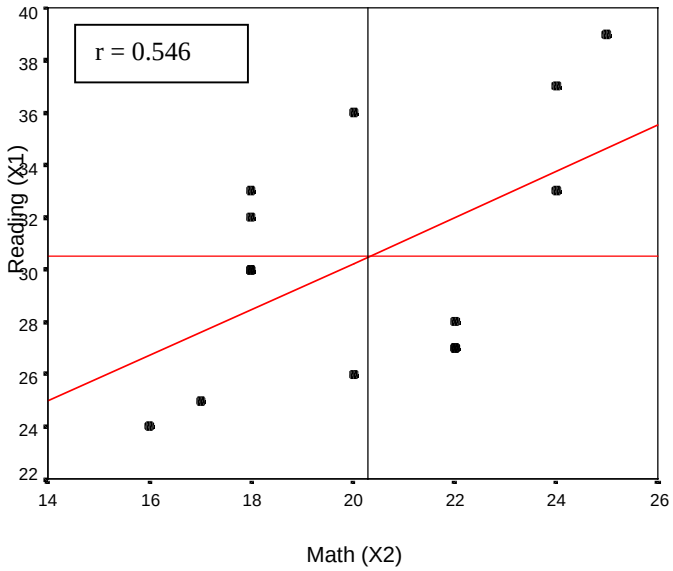
| | Y | X1 | X2 |
|---|---|---|---|
| 1 | 12 | 33 | 18 |
| 2 | 11 | 33 | 24 |
| 3 | 11 | 36 | 20 |
| 4 | 10 | 28 | 22 |
| 5 | 11 | 37 | 24 |
| 6 | 10 | 27 | 22 |
| 7 | 9 | 30 | 18 |
| 8 | 9 | 24 | 16 |
| 9 | 8 | 27 | 22 |
| 10 | 10 | 26 | 20 |
| 11 | 9 | 30 | 18 |
| 12 | 11 | 39 | 25 |
| 13 | 10 | 25 | 17 |
| 14 | 9 | 32 | 18 |



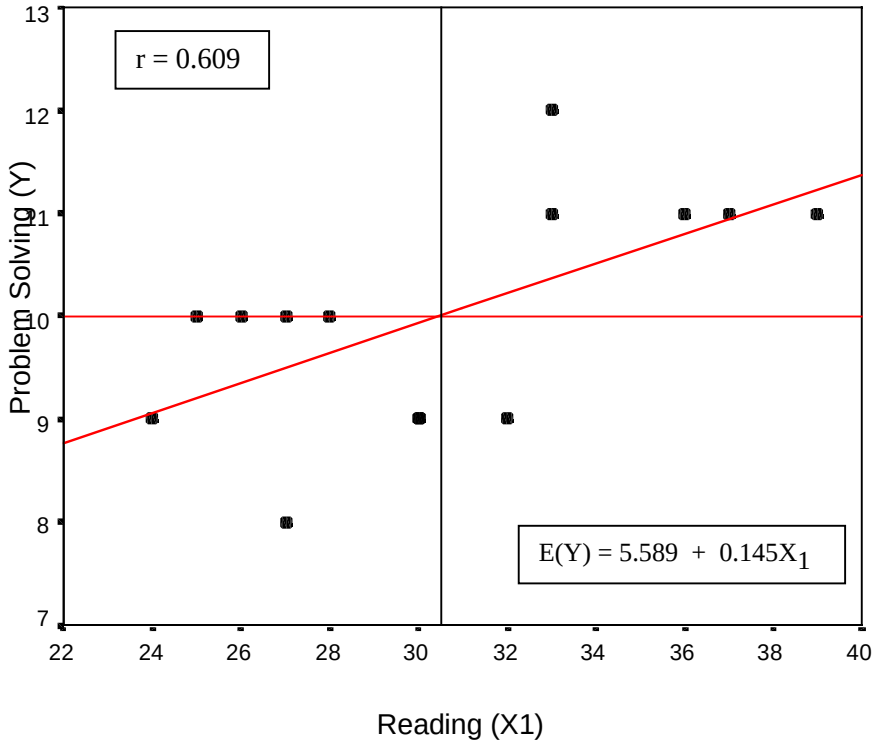

**Descriptive Statistics**

| | Mean | Std. Deviation | N |
|---|---|---|---|
| Problem Solving (Y) | 10.0000 | 1.1094 | 14 |
| Reading (X1) | 30.5000 | 4.6699 | 14 |
| Math (X2) | 20.2857 | 2.8937 | 14 |

**Correlations**

| | | Problem Solving (Y) | Reading (X1) | Math (X2) |
|---|---|---|---|---|
| Problem Solving (Y) | Pearson Correlation | 1.000 | .609 | .359 |
| | Sig. (2-tailed) | . | .021 | .207 |
| | N | 14 | 14 | 14 |
| Reading (X1) | Pearson Correlation | .609 | 1.000 | .546 |
| | Sig. (2-tailed) | .021 | . | .043 |
| | N | 14 | 14 | 14 |
| Math (X2) | Pearson Correlation | .359 | .546 | 1.000 |
| | Sig. (2-tailed) | .207 | .043 | . |
| | N | 14 | 14 | 14 |

* Correlation is significant at the 0.05 level (2-tailed).




b = 0.609(1.1094/4.6699)
b = 0.145

a = 10 – 0.145(30.5)
a = 5.589

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .609[a] | .371 | .318 | .9161 |

a Predictors: (Constant), Reading

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 5.929 | 1 | 5.929 | 7.065 | .021[a] |
| | Residual | 10.071 | 12 | .839 | | |
| | Total | 16.000 | 13 | | | |

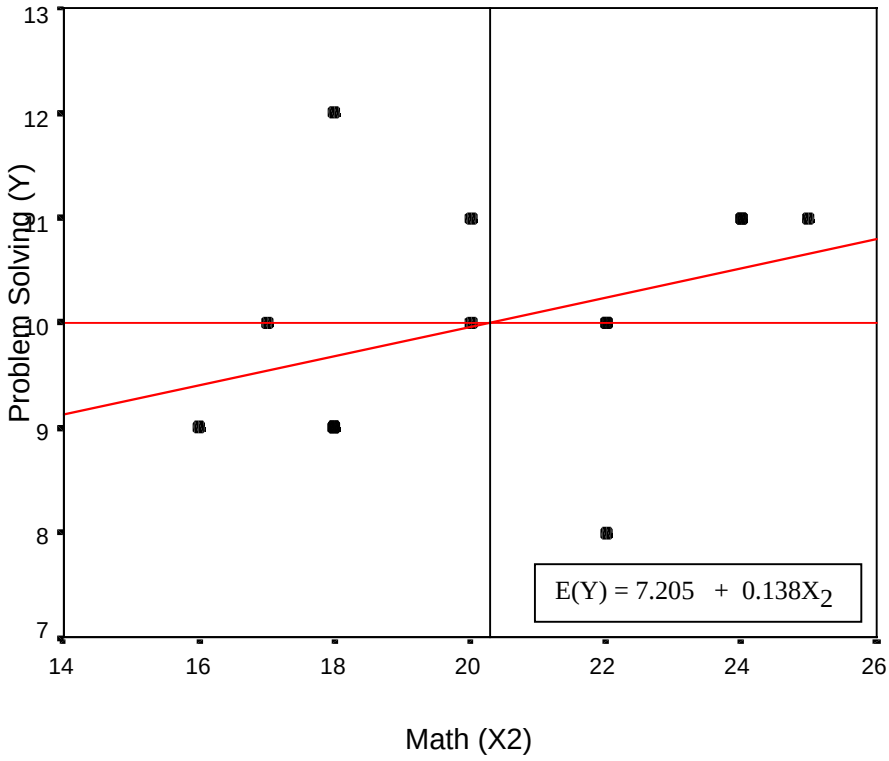
a Predictors: (Constant), Reading
b Dependent Variable: Problem Solving

$R^2 = 5.929/16$
$R^2 = 0.371$

**Coefficients**

| | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | 95% Confidence Interval for B | |
|---|---|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | | Lower Bound | Upper Bound |
| 1 | (Constant) | 5.589 | 1.677 | | 3.332 | .006 | 1.934 | 9.244 |
| | Reading (X1) | .145 | .054 | .609 | 2.658 | .021 | .026 | .263 |

a Dependent Variable: Problem Solving




b = 0.359(1.1094/2.8937)
b = 0.138

a = 10 – 0.138(20.2857)
a = 7.205

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .359[a] | .129 | .057 | 1.0775 |

a Predictors: (Constant), Math

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 2.067 | 1 | 2.067 | 1.780 | .207 |
| | Residual | 13.933 | 12 | 1.161 | | |
| | Total | 16.000 | 13 | | | |

a Predictors: (Constant), Math
b Dependent Variable: Problem Solving

$R^2$ = 2.067/16
$R^2$ = 0.129

**Coefficients**

| | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | 95% Confidence Interval for B | |
|---|---|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | | Lower Bound | Upper Bound |
| 1 | (Constant) | 7.205 | 2.115 | | 3.407 | .005 | 2.597 | 11.812 |
| | Math (X2) | .138 | .103 | .359 | 1.334 | .207 | -.087 | .363 |

a Dependent Variable: Problem Solving

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .610[a] | .372 | .257 | .9560 |

a Predictors: (Constant), Math, Reading

**Correlations**

| | Problem Solving (Y) | Reading (X1) | Math (X2) |
|---|---|---|---|
| Problem Solving (Y) | 1.000 | .609 | .359 |
| Reading (X1) | .609 | 1.000 | .546 |
| Math (X2) | .359 | .546 | 1.000 |

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 5.946 | 2 | 2.973 | 3.253 | .078[a] |
| | Residual | 10.054 | 11 | .914 | | |
| | Total | 16.000 | 13 | | | |

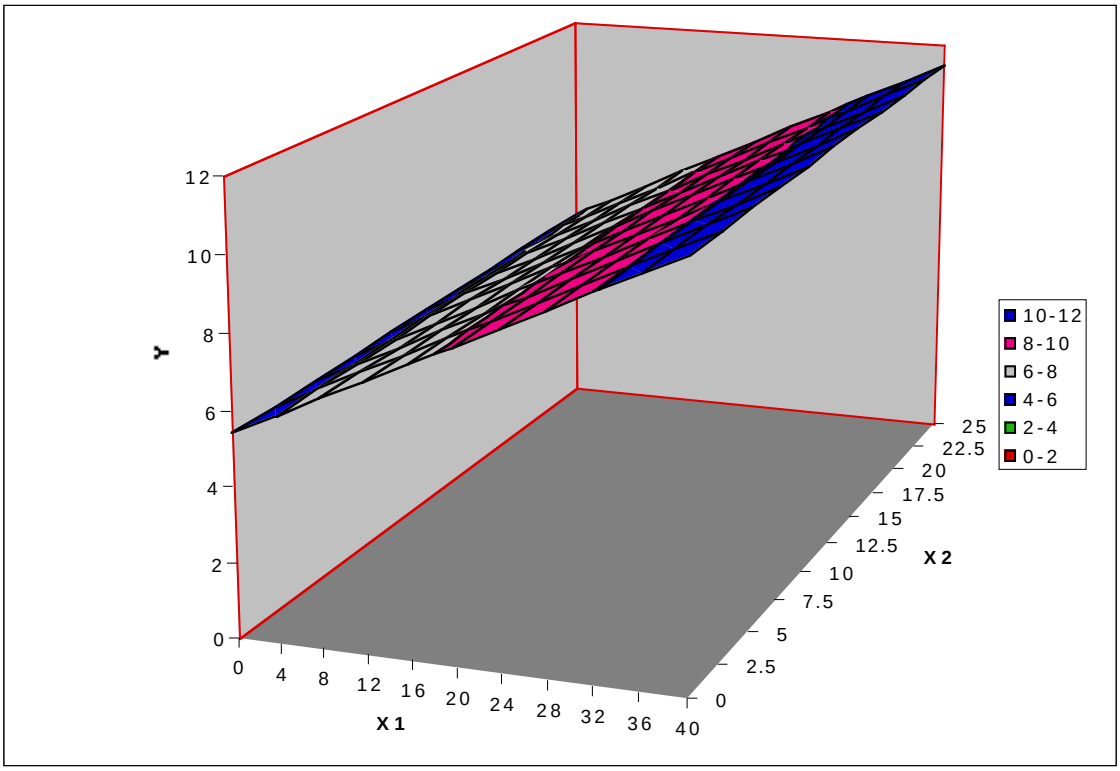
a Predictors: (Constant), Math, Reading
b Dependent Variable: Problem Solving

**Tol = 1-(.546)$^2$=.701**
**VIF = 1/(Tol) = 1.426**

**Coefficients**

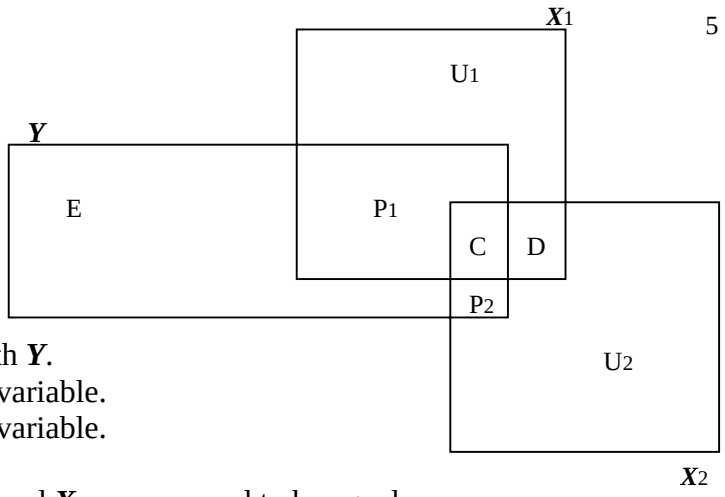
| | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | 95% Confidence Interval for B | | Correlations | | | Collinearity Statistics | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | | | Lower Bound | Upper Bound | Zero-order | Partial | Part | Tol. | VIF |
| 1 (Constant) | 5.443 | 2.062 | | 2.640 | .023 | .905 | 9.982 | | | | | |
| Reading (X1) | .140 | .068 | .588 | 2.060 | .064 | -.010 | .289 | .609 | .528 | .492 | .701 | 1.426 |
| Math (X2) | 1.46E-02 | .109 | .038 | .134 | .896 | -.226 | .255 | .359 | .040 | .032 | .701 | 1.426 |

a Dependent Variable: Problem Solving




$$E(Y) = 5.443 + 0.140X_1 + 0.0146X_2$$

**Venn Diagram for Partitioning Variance in Multiple Regression**




E = Error (or unexplained variation) in ***Y***.
$P_1$ = Explained variation "unique" to $X_1$.
$P_2$ = Explained variation "unique" to $X_2$.
C = Explained variation Common to ***Y***, $X_1$, and $X_2$.
D = Variance shared by $X_1$ and $X_2$, but not shared with ***Y***.
$U_1$ = Variation in $X_1$ that is Unshared with any other variable.
$U_2$ = Variation in $X_2$ that is Unshared with any other variable.

From a standardized solution, the varainces of ***Y***, $X_1$. and $X_2$ are assumed to be equal:

$$Var(Y) = Var(X_1) = Var(X_2) = 1.$$

$Var(Y) = E + P_1 + P_2 + C = 1$. $Var(X_1) = U_1 + P_1 + C + D = 1$. $Var(X_2) = U_2 + P_2 + C + D = 1$.

Full Model $R^2_{Y.12} = \frac{P_1 + P_2 + C}{E + P_1 + P_2 + C} = \frac{P_1 + P_2 + C}{Var(Y)} = 1 - \frac{E}{Var(Y)} =$ **.372**

Zero-order $r^2_{Y1}$ (bivariate) $= \frac{P_1 + C}{E+P_1+P_2+C} = (.609)^2 =$ **.371**

Zero-order $r^2_{Y2}$ (bivariate) $= \frac{P_2 + C}{E+P_1+P_2+C} = (.359)^2 =$ **.129**

Zero-order $r^2_{12}$ (bivariate) $= \frac{C + D}{C+D+P_1+U_1} = (.546)^2 =$ **.298**

First-order Part $r^2_{Y(1.2)} = \frac{P_1}{E+P_1+P_2+C} = (.492)^2 =$ **.242**

Tolerance $= 1 - r^2_{12} = \frac{P_1 + U_1}{C+D+P_1+U_1} =$ **.701**

First-order Part $r^2_{Y(2.1)} = \frac{P_2}{E+P_1+P_2+C} = (.032)^2 =$ **.001**

Common $= R^2_{Y.12} - r^2_{Y(1.2)} - r^2_{Y(2.1)}$
**.372 - .242 - .001 = .129**

**From Previous Analysis**

**$R^2$ = .372**

**Coefficients**

| | Unstandardized Coefficients | | Standardized Coefficients | | | 95% Confidence Interval for B | | Correlations | | | Collinearity Statistics | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | t | Sig. | Lower Bound | Upper Bound | Zero-order | Partial | Part | Tol. | VIF |
| 1 (Constant) | 5.443 | 2.062 | | 2.640 | .023 | .905 | 9.982 | | | | | |
| Reading (X1) | .140 | .068 | .588 | 2.060 | .064 | -.010 | .289 | .609 | .528 | .492 | .701 | 1.426 |
| Math (X2) | 1.46E-02 | .109 | .038 | .134 | .896 | -.226 | .255 | .359 | .040 | .032 | .701 | 1.426 |

a Dependent Variable: Problem Solving

$\Sigma r^2 = .500$ $\Sigma r^2 = .243$

Full Model $R^2$ can also be expressed as: $R^2 = \Sigma\beta r = [(.588)(.609)]+[(.038)(.359)] = .372$

```
COMPUTE predY.1 = 5.589+(.145*x1) .
COMPUTE predY.2 = 7.205+(.138*x2) .
COMPUTE b12 = (4.6699/2.8937)*.546 .
COMPUTE pred1.2 = a12 + (b12*x2) .
COMPUTE b21 = (2.8937/4.6699)*.546 .
COMPUTE pred2.1 = a21 + (b21*x1) .
COMPUTE predY.12 = 5.443 + (.140*x1) + (.0146*x2) .
EXECUTE .
```

```
COMPUTE resY.1 = y - predy.1 .
COMPUTE resY.2 = y - predy.2 .
COMPUTE a12 = 30.50 - (b12*20.2857) .
COMPUTE res1.2 = x1 - pred1.2 .
COMPUTE a21 = 20.2857 - (b21*30.50) .
COMPUTE res2.1 = x2 - pred2.1 .
COMPUTE resY.12 = y - predy.12
```

These correlations are 1 because the predicted value is a linear transformation of the X variable

Multiple $R_{Y\hat{Y}}$ is correlation between Y and $\hat{Y}$

Semi-Partial (Part) Correlations are correlations between Y and Residualized X variables. Unique Contribution to $R^2$; how much each X variable will add to $R^2$ over and above the other Xs.

**Correlations**

| | | Y | X1 | X2 | predY.1 | resY.1 | predY.2 | resY.2 | pred1.2 | res1.2 | pred2.1 | res2.1 | predY.12 | resY.12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y | Pearson Correlation | 1 | .609* | .359 | .609* | .793** | .359 | .933** | .359 | .492 | .609* | .032 | .610* | .793** |
| | Sig. (2-tailed) | . | .021 | .207 | .021 | .001 | .207 | .000 | .207 | .074 | .021 | .914 | .021 | .001 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| X1 | Pearson Correlation | .609* | 1 | .546* | 1.000** | .000 | .546* | .442 | .546* | .837** | 1.000** | .000 | .999** | .000 |
| | Sig. (2-tailed) | .021 | . | .043 | .000 | 1.000 | .043 | .114 | .043 | .000 | .000 | 1.000 | .000 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| X2 | Pearson Correlation | .359 | .546* | 1 | .546* | .034 | 1.000** | .000 | 1.000** | .000 | .546* | .837** | .590* | .000 |
| | Sig. (2-tailed) | .207 | .043 | . | .043 | .909 | . | 1.000 | . | 1.000 | .043 | .000 | .026 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| predY.1 | Pearson Correlation | .609* | 1.000** | .546* | 1 | .000 | .546* | .442 | .546* | .837** | 1.000** | .000 | .999** | .000 |
| | Sig. (2-tailed) | .021 | . | .043 | . | 1.000 | .043 | .114 | .043 | .000 | .000 | 1.000 | .000 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| resY.1 | Pearson Correlation | .793** | .000 | .034 | .000 | 1 | .034 | .837** | .034 | -.022 | .000 | .040 | .002 | .999** |
| | Sig. (2-tailed) | .001 | 1.000 | .909 | 1.000 | . | .909 | .000 | .909 | .940 | 1.000 | .891 | .994 | .000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| predY.2 | Pearson Correlation | .359 | .546* | 1.000** | .546* | .034 | 1 | .000 | 1.000** | .000 | .546* | .837** | .590* | .000 |
| | Sig. (2-tailed) | .207 | .043 | . | .043 | .909 | . | 1.000 | . | 1.000 | .043 | .000 | .026 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| resY.2 | Pearson Correlation | .933** | .442 | .000 | .442 | .837** | .000 | 1 | .000 | .528 | .442 | -.288 | .426 | .849** |
| | Sig. (2-tailed) | .000 | .114 | 1.000 | .114 | .000 | 1.000 | . | 1.000 | .052 | .114 | .317 | .129 | .000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| pred1.2 | Pearson Correlation | .359 | .546* | 1.000** | .546* | .034 | 1.000** | .000 | 1 | .000 | .546* | .837** | .590* | .000 |
| | Sig. (2-tailed) | .207 | .043 | . | .043 | .909 | . | 1.000 | . | 1.000 | .043 | .000 | .026 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| res1.2 | Pearson Correlation | .492 | .837** | .000 | .837** | -.022 | .000 | .528 | .000 | 1 | .837** | -.546* | .808** | .000 |
| | Sig. (2-tailed) | .074 | .000 | 1.000 | .000 | .940 | 1.000 | .052 | 1.000 | . | .000 | .043 | .000 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| pred2.1 | Pearson Correlation | .609* | 1.000** | .546* | 1.000** | .000 | .546* | .442 | .546* | .837** | 1 | .000 | .999** | .000 |
| | Sig. (2-tailed) | .021 | .000 | .043 | .000 | 1.000 | .043 | .114 | .043 | .000 | . | 1.000 | .000 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| res2.1 | Pearson Correlation | .032 | .000 | .837** | .000 | .040 | .837** | -.288 | .837** | -.546* | .000 | 1 | .052 | .000 |
| | Sig. (2-tailed) | .914 | 1.000 | .000 | 1.000 | .891 | .000 | .317 | .000 | .043 | 1.000 | . | .859 | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| predY.12 | Pearson Correlation | .610* | .999** | .590* | .999** | .002 | .590* | .426 | .590* | .808** | .999** | .052 | 1 | .000 |
| | Sig. (2-tailed) | .021 | .000 | .026 | .000 | .994 | .026 | .129 | .026 | .000 | .000 | .859 | . | 1.000 |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| resY.12 | Pearson Correlation | .793** | .000 | .000 | .000 | .999** | .000 | .849** | .000 | .000 | .000 | .000 | .000 | 1 |
| | Sig. (2-tailed) | .001 | 1.000 | 1.000 | 1.000 | .000 | 1.000 | .000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | . |
| | N | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Partial Correlations are correlations between and Residualized Y and Residualized X variables.

Residuals are uncorrelated to the X variables

Structure Coefficients are the correlation between $\hat{Y}$ and the Predictor (X) variables

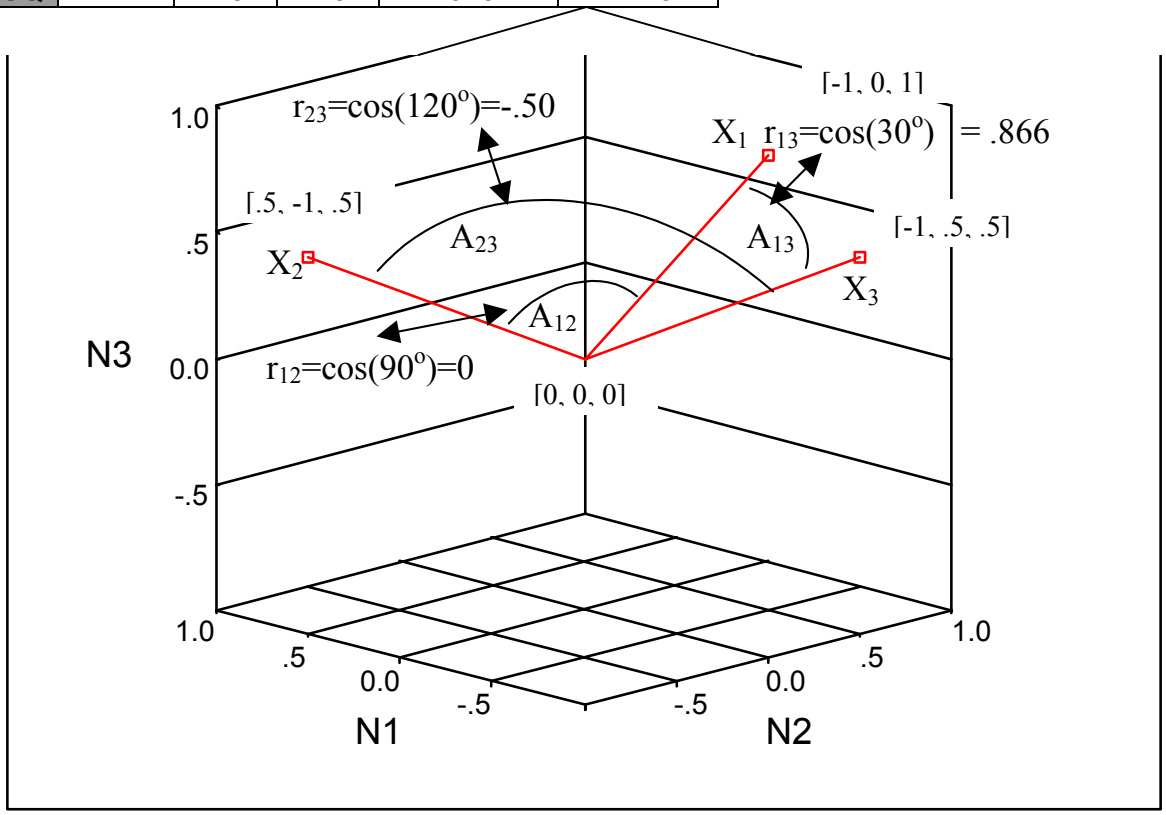
Residuals are uncorrelated to the Predicted Values

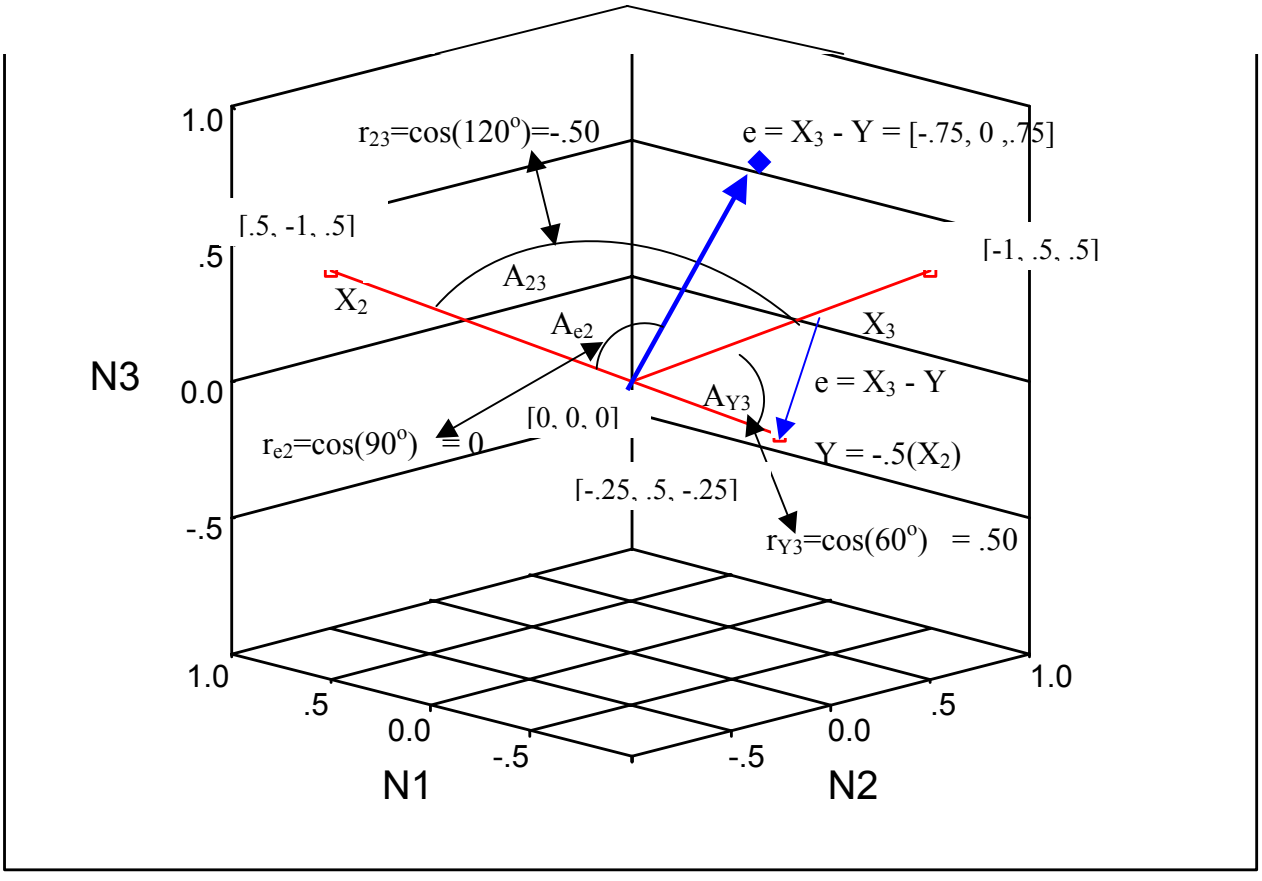
Data

| | X1 | X2 | X3 | Y=-.5(X2) | e=X3-Y |
|---|---|---|---|---|---|
| N1 | -1 | .5 | -1 | -.25 | -.75 |
| N2 | 0 | -1 | .5 | .50 | 0 |
| N3 | 1 | .5 | .5 | -.25 | .75 |
| SSQ | 2 | 1.5 | 1.5 | .375 | 1.125 |

Correlations

| | X1 | X2 | X3 |
|---|---|---|---|
| X1 | 1 | 0 | .866 |
| X2 | 0 | 1 | -.500 |
| X3 | .866 | -.500 | 1 |



**Descriptive Statistics**

| | N | Minimum | Maximum | Mean | Std. Deviation | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| Graduate GPA (Y) | 105 | 1.86 | 4.00 | 3.0908 | .4747 | -.164 | -.726 |
| GRE - Verbal (X1) | 105 | 420 | 620 | 516.67 | 40.66 | .151 | -.205 |
| GRE - Quant (X2) | 105 | 440 | 620 | 517.81 | 38.25 | .083 | -.417 |
| GRE - Analytic (X3) | 105 | 440 | 620 | 521.33 | 39.98 | .012 | -.612 |
| GRE - Psych (X4) | 105 | 430 | 630 | 544.57 | 40.19 | -.269 | .262 |
| College GPA (X5) | 105 | 1.75 | 3.50 | 2.7434 | .3742 | -.130 | -.144 |
| Psych. GPA (X6) | 105 | 1.93 | 4.00 | 3.1679 | .3772 | -.204 | .614 |

←←**NOTE**: These results are taken from the **Descriptives** Module of SPSS NOT the *Descriptives* option under the **Linear Regression** Module.

←←**NOTE**: All variables appear to be fairly symmetric but that does NOT indicate that there are **NO OUTLIERS**.

**Correlations**

| | | Graduate GPA (Y) | GRE - Verb (X1) | GRE - Quant (X2) | GRE - Analyt (X3) | GRE - Psych (X4) | College GPA (X5) | Psych. GPA (X6) |
|---|---|---|---|---|---|---|---|---|
| Pearson Correlation | Graduate GPA (Y) | 1.000 | .394* | .517* | .287* | .119 | .571* | .342* |
| | GRE - Verb. (X1) | .394* | 1.000 | .330* | .295* | .208* | .213* | .141 |
| | GRE - Quant (X2) | .517* | .330* | 1.000 | .687* | .430* | .322* | .415* |
| | GRE - Analy (X3) | .287* | .295* | .687* | 1.000 | .195* | .283* | .226* |
| | GRE - Psych (X4) | .119 | .208* | .430* | .195* | 1.000 | .212* | .180* |
| | College GPA (X5) | .571* | .213* | .322* | .283* | .212* | 1.000 | .372* |
| | Psych. GPA (X6) | .342* | .141 | .415* | .226* | .180* | .372* | 1.000 |
| Sig. (1-tailed) | Graduate GPA (Y) | . | .000 | .000 | .002 | .113 | .000 | .000 |
| | GRE - Verb. (X1) | .000 | . | .000 | .001 | .017 | .014 | .076 |
| | GRE - Quant (X2) | .000 | .000 | . | .000 | .000 | .000 | .000 |
| | GRE - Analy (X3) | .002 | .001 | .000 | . | .023 | .002 | .010 |
| | GRE - Psych (X4) | .113 | .017 | .000 | .023 | . | .015 | .033 |
| | College GPA (X5) | .000 | .014 | .000 | .002 | .015 | . | .000 |
| | Psych. GPA (X6) | .000 | .076 | .000 | .010 | .033 | .000 | . |
| | Graduate GPA (Y) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | GRE - Verb. (X1) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | GRE - Quant (X2) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | GRE - Analy (X3) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | GRE - Psych (X4) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | College GPA (X5) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |
| | Psych. GPA (X6) | 105 | 105 | 105 | 105 | 105 | 105 | 105 |

**NOTE**: The Correlation Matrix (**R**) is symmetric. Thus, the transpose of **R** equals **R**. $\mathbf{R}' = \mathbf{R}$. For example, The first row of **R** is identical to the first column of **R**. The second row of **R** is identical to the second column of **R**. And so forth.

* Correlation is significant at the 0.05 level (1-tailed).

**NOTE:** This is complete case data (or the results from **Listwise** Deletion of Missing Data). If **Pairwise** Deletion were used then the pairwise sample sizes would not necessarily be equal.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .738[a] | .545 | .517 | .3300 |

a Predictors: (Constant), GRE - Verbal (X1), GRE - Quant (X2), GRE - Analytic (X3), GRE - Psych (X4), College GPA (X5), Psych. GPA (X6),

$R^2 = SS_{REG} / SS_{TOT}$
$R^2 = 12.763/23.434 = .545$
$RMSE = \sqrt{.109} = .330$

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 12.763 | 6 | 2.127 | 19.535 | .000[a] |
| | Residual | 10.671 | 98 | .109 | | |
| | Total | 23.434 | 104 | | | |

a Predictors: (Constant), GRE - Verbal (X1), GRE - Quant (X2), GRE - Analytic (X3), GRE - Psych (X4), College GPA (X5), Psych. GPA (X6),
b Dependent Variable: Graduate GPA (Y),

**Example Tolerance for GRE -Verbal (X1)**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .368[a] | .135 | .092 | 38.75 |

a Predictors: (Constant), GRE - Quant (X2), GRE - Analy (X3), GRE - Psych (X4), College GPA (X5), Psych. GPA (X6),
b Dependent Variable: GRE - Verbal (X1),

$X_1 = a + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 + e$

$$Se(B_j) = \frac{(RMSE)\sqrt{VIF}}{Sx_j\sqrt{(N-1)}} \qquad Beta_j = (Sx_j/Sy)B$$

$Se(B_1) = \frac{(.33)(1.12)}{(40.66)(10.2)} = \mathbf{.001}$ $\qquad Beta_1 = \frac{(40.66)(.0027)}{(.4747)} = \mathbf{.231}$

**Tolerance = 1 - $R^2$ = 1 -.135 = .865**
**VIF = 1/Tolerance = 1/.865 = 1.156**

**Coefficients**

| | Unstandardized Coefficients | | Standardized Coefficients | | | 95% Confidence Interval for B | | Correlations | | | Collinearity Statistics | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | t | Sig. | Lower Bound | Upper Bound | Zero-order | Partial | Part | Tol. | VIF |
| 1 (Constant) | **-.5350** | **.620** | | **-.862** | **.391** | **-1.766** | **.696** | | | | | |
| **GRE - Verb (X1)** | **.0027** | **.001** | **.231** | **3.155** | **.002** | **.001** | **.004** | **.394** | **.304** | **.215** | **.865** | **1.156** |
| **GRE - Quant (X2)** | **.0070** | **.001** | **.560** | **5.070** | **.000** | **.004** | **.010** | **.517** | **.456** | **.346** | **.380** | **2.630** |
| **GRE - Analy (X3)** | **-.0030** | **.001** | **-.254** | **-2.631** | **.010** | **-.005** | **-.001** | **.287** | **-.257** | **-.179** | **.497** | **2.013** |
| **GRE - Psych (X4)** | **-.0026** | **.001** | **-.218** | **-2.826** | **.006** | **-.004** | **-.001** | **.119** | **-.275** | **-.193** | **.781** | **1.280** |
| **College GPA (X5)** | **.5810** | **.097** | **.458** | **6.003** | **.000** | **.389** | **.773** | **.571** | **.518** | **.409** | **.797** | **1.254** |
| **Psych. GPA (X6)** | **.0039** | **.099** | **.003** | **.040** | **.969** | **-.192** | **.200** | **.342** | **.004** | **.003** | **.752** | **1.330** |

a Dependent Variable: Graduate GPA (Y),

$(\text{Part Corr}_j)^2 = R^2_{FULL} - R^2_{(\text{Model} - X_j)}$
$(\text{Part Corr}_1)^2 = .545 - .498 = .047$
$(\text{Part Corr}_1) = \sqrt{.047} \approx .215$
Differences due to rounding error.

**Example Part Correlation for GRE -Verbal (X1)**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .706[a] | .498 | .573 | .3446 |

a Predictors: (Constant), GRE - Quant (X2), GRE - Analytic (X3), GRE - Psych (X4), College GPA (X5), Psych. GPA (X6),
b Dependent Variable: Graduate GPA,

$Y = a + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 + e$

**Residual Statistics[a]**

| | Minimum | Maximum | Mean | Std. Deviation | N |
|---|---|---|---|---|---|
| Predicted Value | 2.3084 | 3.8635 | 3.0908 | .3503 | 105 |
| Standardized Predicted Value | -2.2330 | 2.2060 | .000 | 1.000 | 105 |
| Standard Error of Predicted Value | .04165 | .1270 | .08326 | .01819 | 105 |
| Adjusted Predicted Value | 2.3477 | 3.8733 | 3.0924 | .3496 | 105 |
| Residual | -.7803 | .8377 | -2.03E-16 | .3203 | 105 |
| Standardized Residual | -2.365 | 2.539 | .000 | .971 | 105 |
| Studentized Residual | -2.436 | 2.570 | -.002 | .999 | 105 |
| Deleted Residual | -.8278 | .8589 | -.00163 | .3395 | 105 |
| Studentized Deleted Residual | -2.500 | 2.648 | -.003 | 1.009 | 105 |
| Mahalanobis Distance | .666 | 14.409 | 5.943 | 2.937 | 105 |
| Cook's Distance | .000 | .067 | .009 | .012 | 105 |
| Centered Leverage Value | .006 | .139 | .057 | .028 | 105 |

a Dependent Variable: Graduate GPA (Y),

**Case 45 has:**

| | | | |
|---|---|---|---|
| GradGPA = | 2.21 | DFBETA(Intercept) = | -.07241 |
| GRE-V = | 520 | DFBETA(GRE-V) = | -.00010 |
| GRE-Q = | 470 | DFBETA(GRE-Q) = | .00039 |
| GRE-A = | 490 | DFBETA(GRE-A) = | -.00005 |
| GRE-Psy = | 510 | DFBETA(GRE-Psy) = | .00006 |
| CollGPA = | 2.62 | DFBETA(CollGPA) = | .00951 |
| PsychGPA = | 3.55 | DFBETA(PsyGPA) = | -.03662 |

←← **NOTE**: The Fit is better in terms of $R^2$, Adjusted $R^2$, and RMSE

## Analysis with Case 45 excluded
## Compare Results with Previous Analysis

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .742[a] | .550 | .522 | .3242 |

a Predictors: (Constant), GRE - Verbal (X1), GRE - Quant (X2), GRE - Analytic (X3), GRE - Psych (X4), College GPA (X5), Psych. GPA (X6),

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 12.458 | 6 | 2.076 | 19.757 | .000[a] |
| | Residual | 10.194 | 97 | .105 | | |
| | Total | 22.651 | 103 | | | |

←← **NOTE**: The Residual and Total *df*s are reduced by 1.

b Dependent Variable: Graduate GPA (Y),

**Coefficients**

| | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. | 95% Confidence Interval for B | | Correlations | | | Collinearity Statistics | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | | | Lower Bound | Upper Bound | Zero-order | Partial | Part | Tol. | VIF |
| 1 **(Constant)** | **-.4620** | **.610** | | **-.758** | **.450** | **-1.674** | **.749** | | | | | |
| **GRE-V (X1)** | **.0028** | **.001** | **.244** | **3.329** | **.001** | **.001** | **.004** | **.402** | **.320** | **.227** | **.862** | **1.160** |
| **GRE-Q (X2)** | **.0066** | **.001** | **.534** | **4.831** | **.000** | **.004** | **.009** | **.507** | **.440** | **.329** | **.379** | **2.637** |
| **GRE - A (X3)** | **-.0030** | **.001** | **-.254** | **-2.635** | **.010** | **-.005** | **-.001** | **.278** | **-.258** | **-.180** | **.499** | **2.002** |
| **GRE-Psych (X4)** | **-.0026** | **.001** | **-.226** | **-2.946** | **.004** | **-.004** | **-.001** | **.106** | **-.287** | **-.201** | **.786** | **1.273** |
| **College GPA (X5)** | **.5721** | **.095** | **.458** | **6.004** | **.000** | **.383** | **.761** | **.575** | **.520** | **.409** | **.796** | **1.256** |
| **Psych. GPA (X6)** | **.0405** | **.099** | **.033** | **.411** | **.682** | **-.155** | **.236** | **.368** | **.042** | **.028** | **.736** | **1.358** |

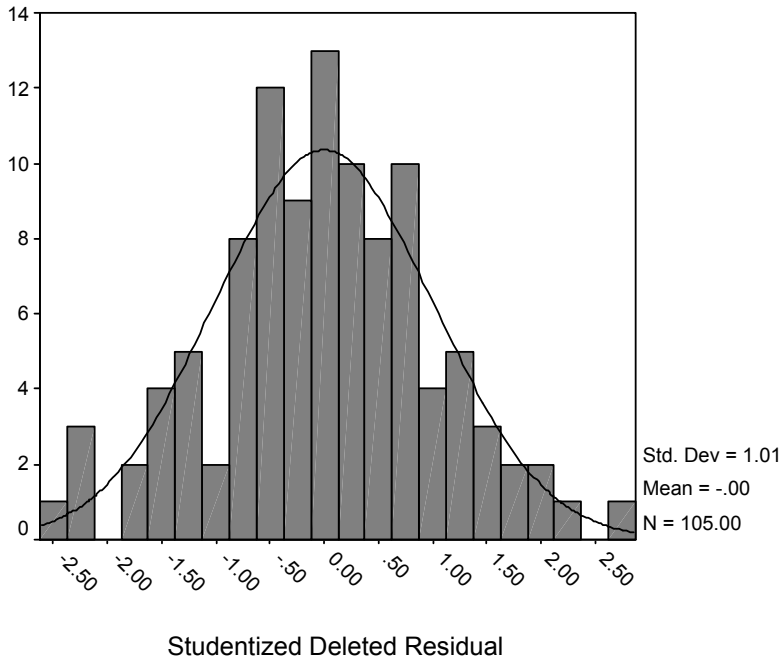
a Dependent Variable: Graduate GPA (Y),

**NOTE**: B coefficients are changed by the respective DFBETA values. For example,

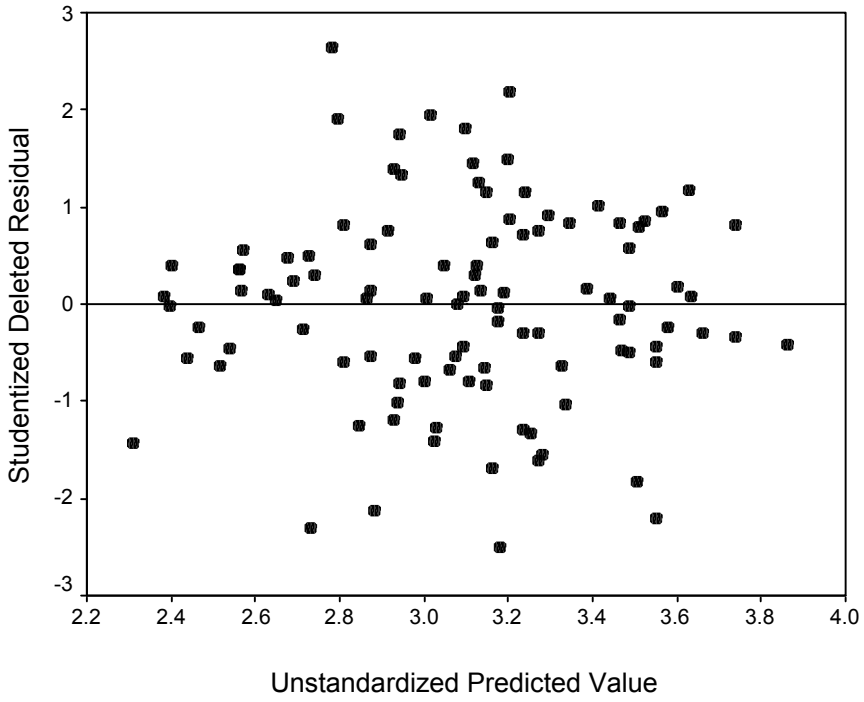
B(PsychGPA) = .0039 in the previous analysis.

B(PsychGPA) = .0405 with case 45 excluded.

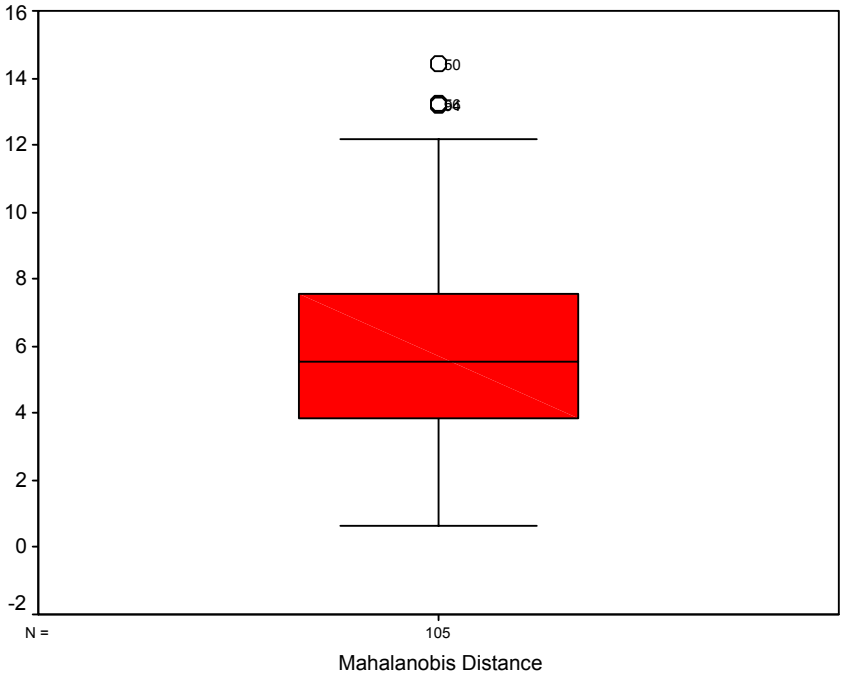
DFBETA(PsychGPA) = -.0366 for case 45.



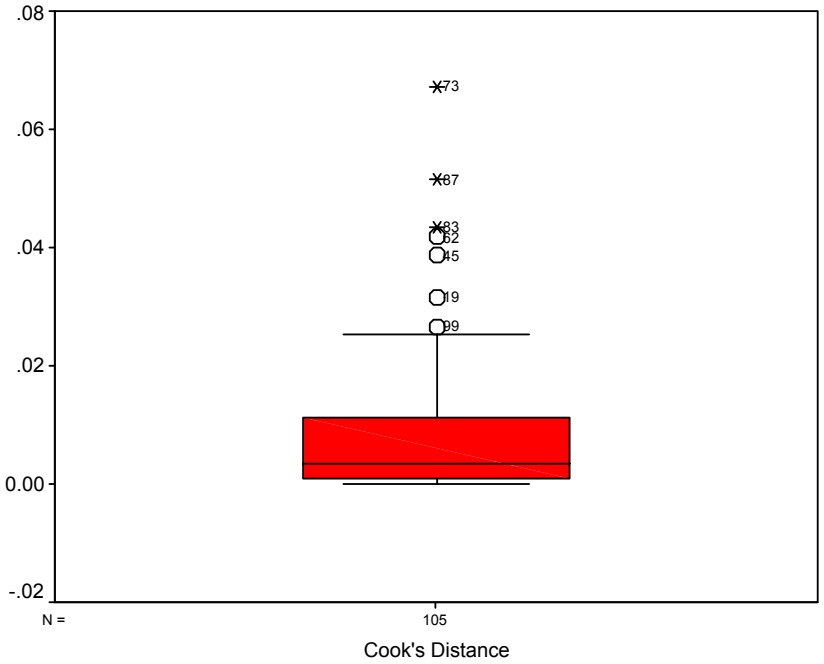

**Analysis of Normality of Errors Distributed as $t_{(N-k-1)}$**



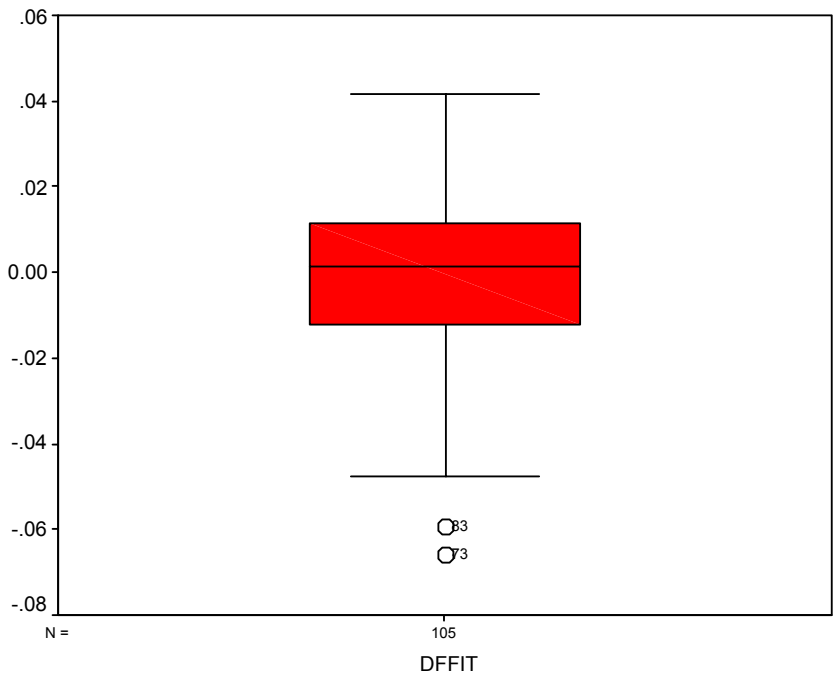

**Analysis of Multivariate Homoscedasticy**



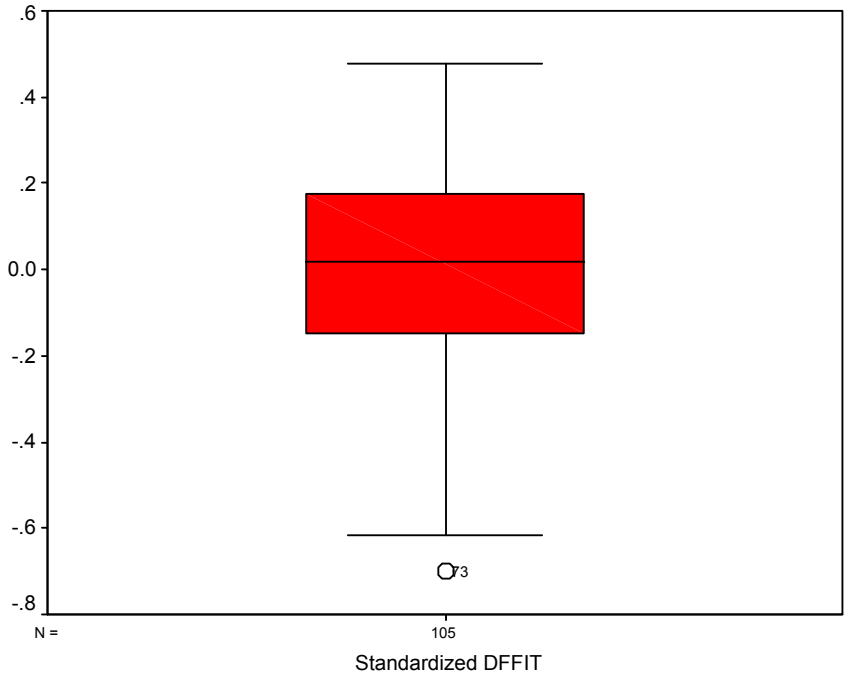

**Global Index for Multivariate Outliers**
**Derived from the Hat Matrix (Leverage)**



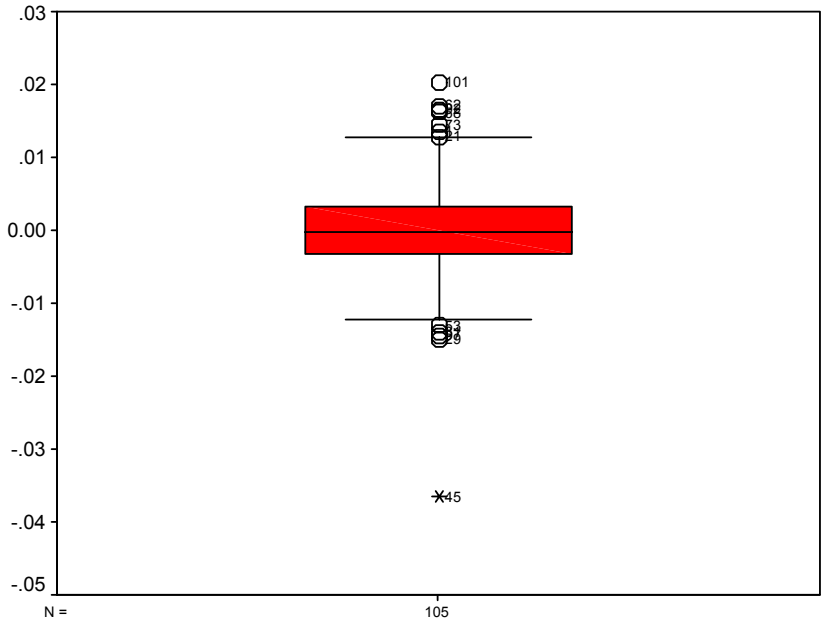

**Global Index for Multivariate Influence**
**How each value changes all *b* coefficients**



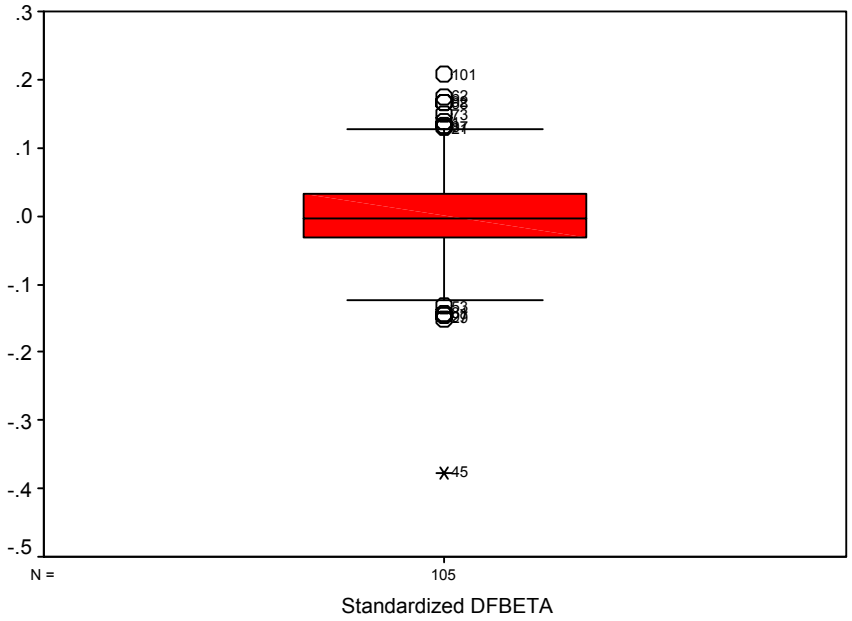

**Global Index for Multivariate Influence**
**How each value changes its Predicted Value**



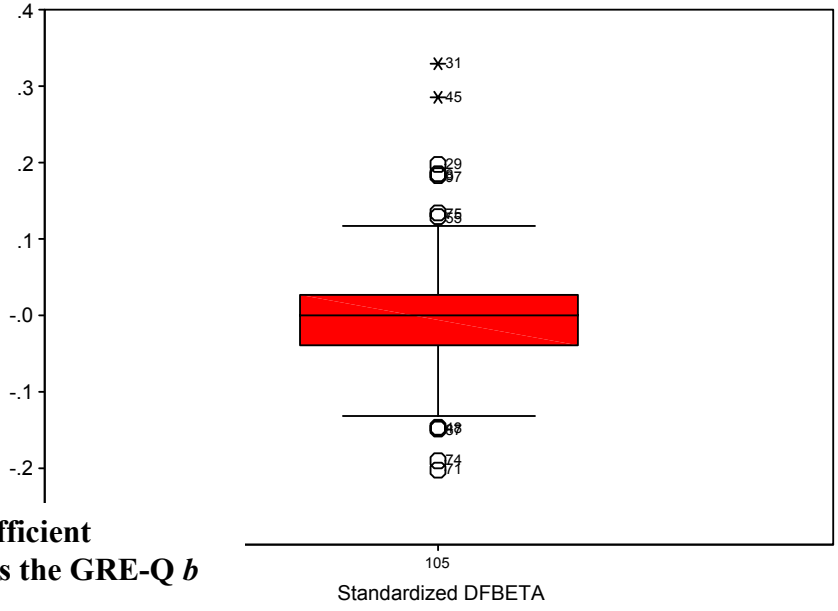

**Global Index for Multivariate Influence**
**How each value changes its Predicted Value**
**(Standardized)**




**Influence index for the Psych GPA b coefficient**
**How each value changes the Psych GPA *b***




**Influence index for the Psych GPA b coefficient**
**Z-score for How each value changes the Psych GPA *b***




**Influence index for the GRE-Q coefficient**
**Z-score for How each value changes the GRE-Q *b***

**Re-analysis with Psych. GPA (non-significant), GRE-Psych (negative value, nonsensical), and GRE-Analytic (negative value, nonsensical) Removed from the Model**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .696[a] | .485 | .469 | .3458 |

a Predictors: (Constant), GRE - Verbal (X1), GRE - Quant (X2), College GPA (X5),

$R^2 = SS_{REG} / SS_{TOT}$
$R^2 = 11.360/23.434 = .485$
$RMSE = \sqrt{.120} = .3458$

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 11.360 | 3 | 3.787 | 31.673 | .000[a] |
| | Residual | 12.075 | 101 | .120 | | |
| | Total | 23.434 | 104 | | | |

a Predictors: (Constant), GRE - Verbal (X1), GRE - Quant (X2), College GPA (X5),
b Dependent Variable: Graduate GPA (Y),

**Example Tolerance for GRE -Verbal (X1)**

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .349[a] | .122 | .105 | 38.47 |

a Predictors: (Constant), GRE - Quant (X2), , College GPA (X5),
b Dependent Variable: GRE - Verbal (X1),
$X_1 = a + b_2X_2 + b_5X_5 + e$

$Se(B_j) = \frac{(RMSE)\sqrt{VIF}}{Sx_j\sqrt{(N-1)}}$ $\quad Beta_j = (Sx_j/Sy)B$

$Se(B_1) = \frac{[(.346)(1.144]}{[(40.66)(10.2)]} = \mathbf{.001}$ $\quad Beta_1 = \frac{(40.66)(.0023)}{(.4747)} = \mathbf{.199}$

**Tolerance = 1 - $R^2$ = 1 -.122 = .878**
**VIF = 1/Tolerance = 1/.878 = 1.139**

**Coefficients**

| | Unstandardized Coefficients | | Standardized Coefficients | | | 95% Confidence Interval for B | | Correlations | | | Collinearity Statistics | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B | Std. Error | Beta | t | Sig. | Lower Bound | Upper Bound | Zero-order | Partial | Part | Tol. | VIF |
| 1 **(Constant)** | **-1.615** | **.552** | | **-2.926** | **.004** | **-2.709** | **-.520** | | | | | |
| **GRE - Verb (X1)** | **.0023** | **.001** | **.199** | **2.617** | **.010** | **.001** | **.004** | **.394** | **.252** | **.187** | **.878** | **1.139** |
| **GRE - Quant (X2)** | **.0039** | **.001** | **.314** | **3.986** | **.000** | **.002** | **.006** | **.517** | **.369** | **.285** | **.825** | **1.213** |
| **College. GPA (X5)** | **.5420** | **.096** | **.427** | **5.626** | **.000** | **.351** | **.733** | **.571** | **.488** | **.402** | **.884** | **1.132** |

a Dependent Variable: Graduate GPA (Y),

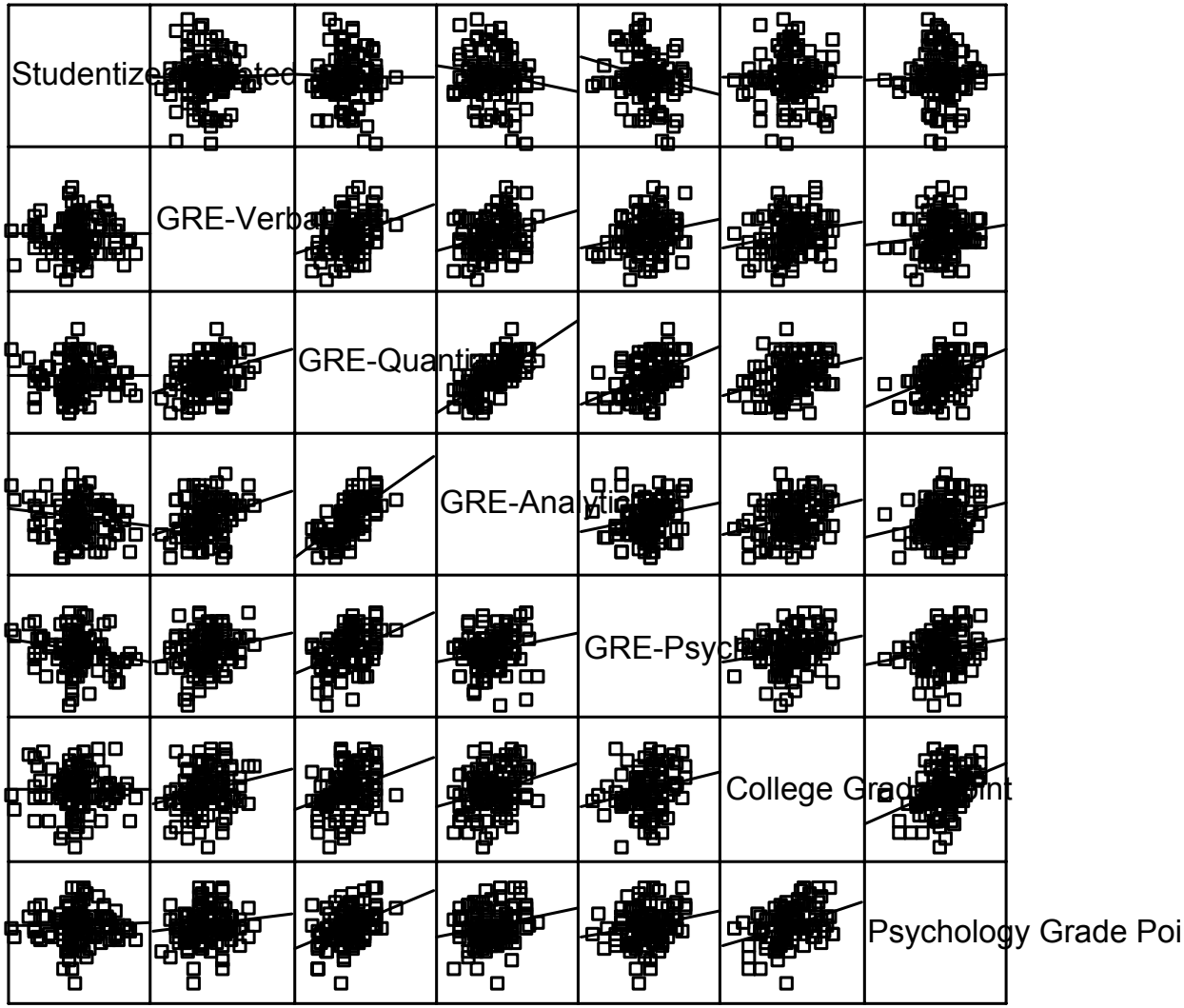
$(\text{Part Corr}_j)^2 = R^2_{FULL} - R^2_{(Model - X_j)}$
$(\text{Part Corr}_1)^2 = .485 - .450 = .035$
$(\text{Part Corr}_1) = \sqrt{.035} = .187$

**Example Part Correlation for GRE -Verbal (X1)**

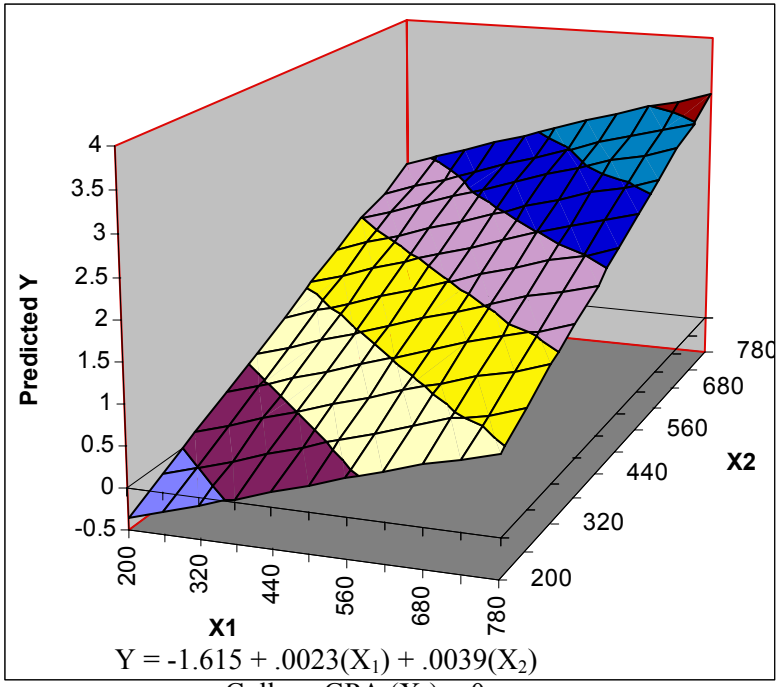
**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .671[a] | .450 | .439 | .3555 |

a Predictors: (Constant), GRE - Quant (X2), College GPA (X5),
b Dependent Variable: Graduate GPA,
$Y = a + b_2X_2 + b_5X_5 + e$

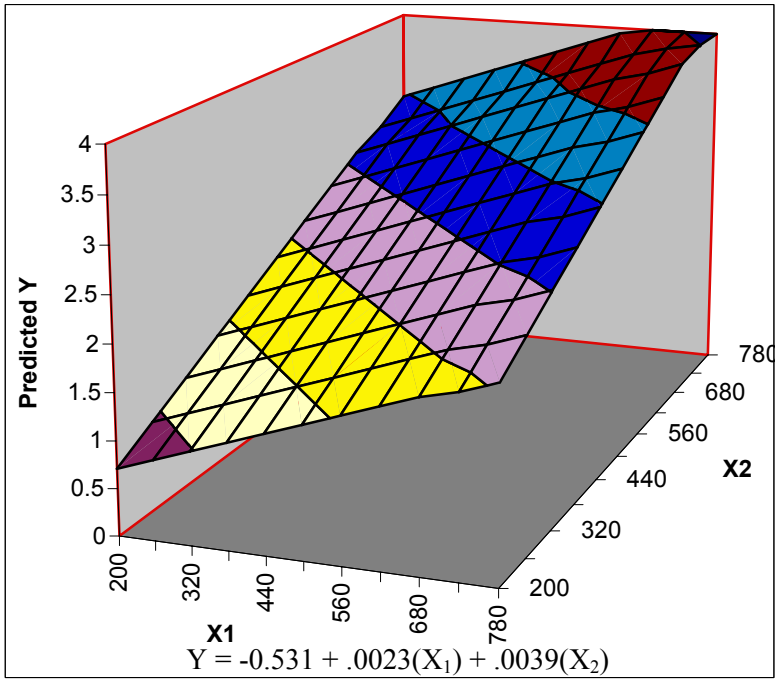



Studentized Deleted Residuals from the 3-predictor model plotted against all X variables. The plots in columns 2, 3, and 6 for the first row show NO indication of non-linearity for the 3 predictors actually in the model. Plots in columns 4 and 5 of the first row show that GRE-A and GRE-Psy are correlated to the Residual and thus could added added to the model. They were excluded because the coefficients were negative. The last plot (column 7) in the first row shows tha Psych-GPA is not correlated to the residual and should not be added to the model.



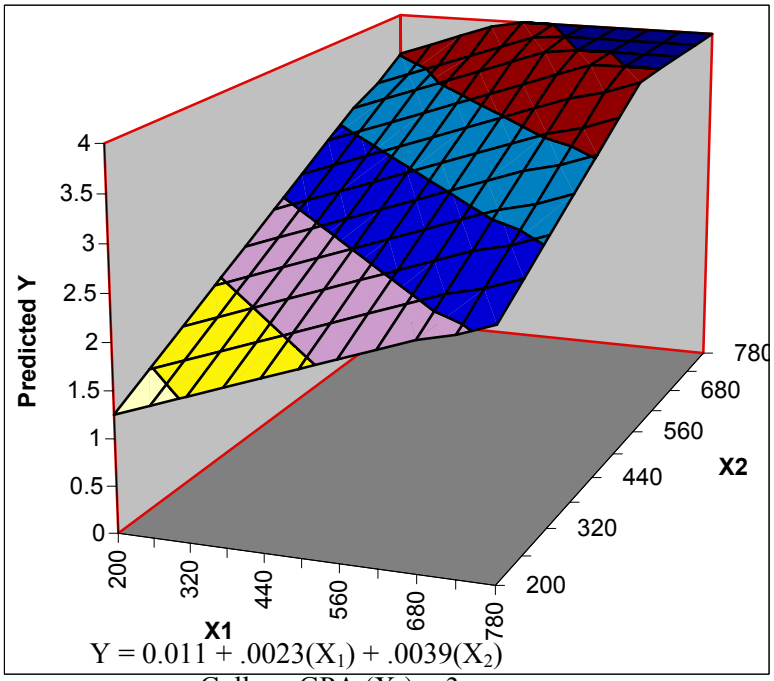

$Y = -1.615 + .0023(X_1) + .0039(X_2)$

College GPA ($X_5$) = 0



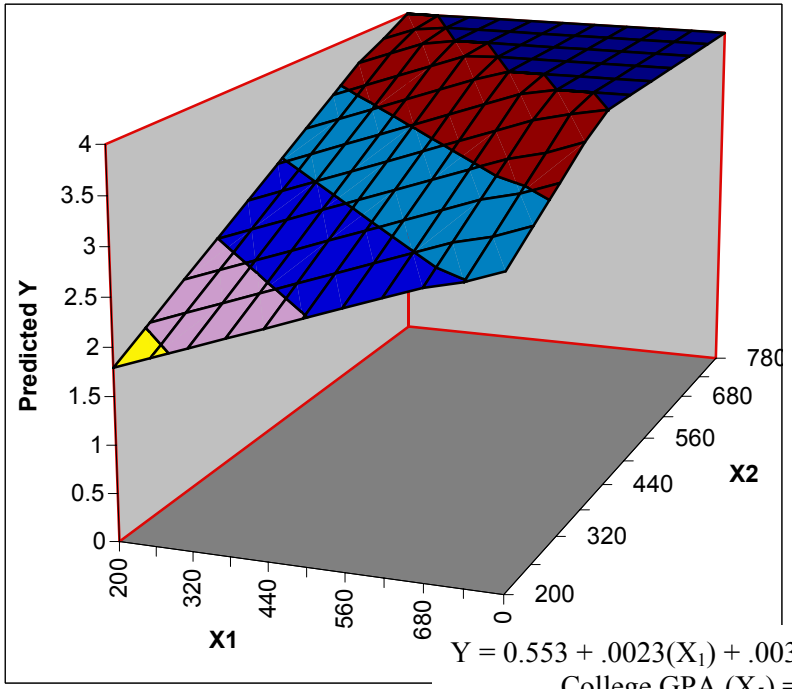

$Y = -0.531 + .0023(X_1) + .0039(X_2)$

College GPA ($X_5$) = 2




$Y = 0.011 + .0023(X_1) + .0039(X_2)$

College GPA ($X_5$) = 3




$Y = 0.553 + .0023(X_1) + .0039(X_2)$

College GPA ($X_5$) = 4

```
proc reg data=gradgpa ;
model ggpa=grev greq greanal grepsy cgpa psygpa  /clb stb ss1 ss2 scorr1  scorr2; run;
```

The REG Procedure
Dependent Variable: ggpa

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 12.76316 | 2.12719 | 19.54 | <.0001 |
| Error | 98 | 10.67118 | 0.10889 | | |
| Corrected Total | 104 | 23.43434 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.32998 | R-Square | 0.5446 |
| Dependent Mean | 3.09076 | Adj R-Sq | 0.5168 |
| Coeff Var | 10.67647 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| | Standardized Estimate | 95% Confidence Limits | |
|---|---|---|---|---|---|---|---|---|
| Intercept | 1 | -0.53489 | 0.62033 | -0.86 | 0.3906 | 0 | -1.76592 | 0.69613 |
| grev | 1 | 0.00270 | 0.00085577 | 3.15 | 0.0021 | 0.23124 | 0.00100 | 0.00440 |
| greq | 1 | 0.00695 | 0.00137 | 5.07 | <.0001 | 0.56047 | 0.00423 | 0.00968 |
| greanal | 1 | -0.00302 | 0.00115 | -2.63 | 0.0099 | -0.25450 | -0.00530 | -0.00074283 |
| grepsy | 1 | -0.00257 | 0.00091097 | -2.83 | 0.0057 | -0.21801 | -0.00438 | -0.00076697 |
| cgpa | 1 | 0.58131 | 0.09684 | 6.00 | <.0001 | 0.45824 | 0.38912 | 0.77349 |
| psygpa | 1 | 0.00391 | 0.09893 | 0.04 | 0.9686 | 0.00311 | -0.19241 | 0.20023 |

| Variable | **Type I SS** | **Type II SS** | Squared Semi-partial Corr Type I | **Squared Semi-partial Corr Type II** | | Type 1 Sequential | **Type 2,3, 4 Last Order** |
|---|---|---|---|---|---|---|---|
| Intercept | 1003.04496 | 0.08096 | . | . | Order | | |
| grev | **3.64154** | **1.08368** | **0.15539** | **0.04624** | **0** | **3.64154/23.43** | **1.08368/23.43** |
| greq | **3.93417** | **2.79914** | **0.16788** | **0.11945** | **1** | **3.93417/23.43** | **2.79914/23.43** |
| greanal | **0.32813** | **0.75391** | **0.01400** | **0.03217** | **2** | **3.64154/23.43** | **0.75391/23.43** |
| grepsy | **0.55809** | **0.86986** | **0.02381** | **0.03712** | **3** | **0.55809/23.43** | **0.86986/23.43** |
| cgpa | **4.30106** | **3.92336** | **0.18354** | **0.16742** | **4** | **4.30106/23.43** | **3.92336/23.43** |
| psygpa | **0.00016999** | **0.00016999** | **0.00000725** | **0.00000725** | **5** | **0.00017/23.43** | **0.00017/23.43** |

Type 1 SS/SS Total **Type 2 SS/SST**
= Semi-Parital Square Correlations