T. Mark Beasley **Split-Plot ANOVA handout**

In the Split-Plot ANOVA, three factors represent separate sources of variance. Two interactions also present independent sources of variation. Suppose a design in which a Factor A has two levels (e.g., Treatment vs. Control) and Factor B has three levels of a Repeated Measures factor (e.g., Baseline, One Month Three Months). The design has 2x3=6 cell means.

Condition	Baseline (B_1)	1 Month (B_2)	3 Months (B_3)	$\bar{Y}_{P*}$
Control A_1	8	9	13	10
	8	8	14	10
	7	8	12	9
	6	8	10	8
	6	7	11	8
$\bar{Y}_{AB}$	$\bar{Y}_{11} = 7$	$\bar{Y}_{12} = 8$	$\bar{Y}_{13} = 12$	$\bar{Y}_{1*} = 9$
Treatment A_2	8	17	8	11
	7	18	8	11
	7	16	7	10
	7	14	6	9
	6	15	6	9
$\bar{Y}_{AB}$	$\bar{Y}_{21} = 7$	$\bar{Y}_{22} = 16$	$\bar{Y}_{23} = 7$	$\bar{Y}_{2*} = 10$
	$\bar{Y}_{*1} = 7$	$\bar{Y}_{*2} = 12$	$\bar{Y}_{*3} = 9.5$	$\bar{Y}_{**} = 9.5$

ANOVA MODEL: $Y_{ijk} = \mu_{**} + \alpha_j + \pi_{i(j)} + \beta_k + \alpha\beta_{jk} + \beta\pi_{ik(j)} + \varepsilon_{ijk}$

Factor A has two marginal means, $\bar{Y}_{1*}$ and $\bar{Y}_{2*}$, and (A-1=2-1=1) degree of freedom. The null hypothesis for Factor A is H_0: $\mu_{1*} = \mu_{2*}$ or H_0: $\Sigma\alpha^2_j = 0$.

Because Factor A is a Between-Subjects factor the Sum of Squares of nested subject effect ($\Sigma\pi^2_{i(j)}$) is a source of error.

Factor B has three marginal means, $\bar{Y}_{*1}$, $\bar{Y}_{*2}$, and $\bar{Y}_{*3}$, and (B-1=3-1=2) degrees of freedom. The null hypothesis for Factor B is H_0: $\mu_{*1} = \mu_{*2} = \mu_{*3}$ or H_0: $\Sigma\beta^2_k = 0$.

The interaction term is multiplicative conceptually; thus, the AxB interaction has (A-1)x(B-1) degrees of freedom. The null hypothesis quite complex,

H_0: $(\mu_{11} - \mu_{21}) = (\mu_{12} - \mu_{22}) = (\mu_{13} - \mu_{23})$ or H_0: $\Sigma\alpha\beta^2_{jk} = 0$.

It implies that the absence of an interaction indicates that the main effects of Factor A are independent of the main effects of Factor B. Since interactions are symmetric, the absence of an interaction also indicates that the main effects of Factor B are independent of the main effects of Factor A.

Because both the Repeated Measures main effect (Factor B) and the AxB Interaction involve the within-subjects factor, the Sum of Squares for the Repeated Measures by Subjects nested in groups interaction ($\Sigma\beta\pi^2_{ik(j)}$) is a source of error.

The computation of the Sums of Squares (SS) for the main effects of Factors A and B are similar to the two-way analysis. Marginal means are subtracted from the grand mean, squared, weighted by the marginal sample size, and summed.

Computations

Split-Plot ANOVA Source Table: $Y_{ijk} = \mu^{**} + \alpha_j + \pi_{i(j)} + \beta_k + \alpha\beta_{jk} + \beta\pi_{ik(j)} + \varepsilon_{ijk}$

Source		Sum of Squares	df	Mean Square	F
BETWEEN- SUBJECTS					
Factor A	$(\Sigma\alpha^2_j)$ a_1 = -0.5 a_2 = 0.5	$\Sigma n_A(\overline{Y}_{A*} - \overline{Y}_{**})^2$ $15(9-9.5)^2$ $+ 15(10-9.5)^2$ = **7.5**	$(A - 1)$ (2 - 1) = **1**	$SS_A/(A-1)$ 7.5/1 = **7.5**	MS_A/MS_P 7.5/3 = **2.50**
Factor P	$(\Sigma\pi^2_{i(j)})$ p_1 = 1 . . . p_5 = -1 p_6 = -1 . . . p_{10} = 1	$\Sigma B(\overline{Y}_{P*} - \overline{Y}_{A*})^2$ $3(10-9)^2$ . . . $+ 3(8-9)^2$ $+ 3(11-10)^2$ . . . $+ 3(9-10)^2$ = **24**	$(N - A)$ (10-2) = **8**	$SS_P/(N-A)$ 24/8=**3**	
WITHIN- SUBJECTS					
Factor B	$(\Sigma\beta^2_k)$ b_1 = -2.5 b_2 = 2.5 b_3 = 0	$\Sigma n_B(\overline{Y}_{*B} - \overline{Y}_{**})^2$ $10(7-9.5)^2$ $+ 10(12-9.5)^2$ $+ 10(9.5-9.5)^2$ = **125**	$(B - 1)$ (3 - 1) = **2**	$SS_B/(B-1)$ 125/2=**62.5**	MS_B/MS_{BxP} 62.5/0.5 = **125.00**
Interaction (AxB)	$(\Sigma\alpha\beta^2_{jk})$ ab_{11} = 0.5 ab_{21} = -0.5 ab_{12} = -3.5 ab_{22} = 3.5 ab_{13} = 3.0 ab_{23} = -3.0	$\Sigma n_{AB}(\overline{Y}_{AB} - \overline{Y}_{A*} - \overline{Y}_{*B} + \overline{Y}_{**})^2$ $5(7-9-7+9.5)^2$ $+ 5(7-10-7+9.5)^2$ $+ 5(8-9-12+9.5)^2$ $+ 5(16-10-12+9.5)^2$ $+ 5(12-9-9.5+9.5)^2$ $+ 5(7-10-9.5+9.5)^2$=**215**	$(A-1)(B-1)$ (2-1)(3-1) (1)(2) = **2**	SS_{AB}/df_{AB} 215/2 = **107.50**	MS_{AB}/MS_{BxP} 107.5/0.5 = **215.00**
Within-Subjects (Error)	$(\Sigma\beta\pi^2_{ik(j)})$ bp_{11} = 0 . . . bp_{53} = 0 bp_{61} = 0 . . . $bp_{10,3}$ = 0	$\Sigma(Y_{ijk} - \overline{Y}_{P*} - \overline{Y}_{AB} + \overline{Y}_{A*})^2$ $(8-10-7+9)^2$ . . . $+ (11-8-12+9)^2$ $+ (8-11-7+10)^2$ . . . $+ (6-9-7+10)^2$ = **8**	$(B-1)(N-A)$ (3-1)(10-2) **16**	SS_W/df_W (8/16	= **0.5**)
Total Variance		$\Sigma(Y_i - \overline{Y}_{**})^2$ = **379.5**	$BN - 1$ 3(10)-1=**29**	($s^2 = S_T/BN-1$	= **13.09**)

where N = total number of subjects, A= number of groups for Factor A, B= number of measures for Factor B, $\overline{Y}_{**}$ = the grand mean of Y across all observations, Y_{ijk} = each individual score on Y, $\overline{Y}_{P*}$ = the mean for each subject, n_A = the number of cases in each group of Factor A, n_B = the number of cases in each measure of Factor B, and n_{AB} = the number of cases in AB cell.

Reporting the Results and Simple Effects Analysis

Condition	Time of Testing: Baseline	One Month	3 Months
CONTROL			
Mean	7.00	8.00	12.00
SD	1.00	0.71	1.58
TREATMENT			
Mean	7.00	16.00	7.00
SD	0.71	1.58	1.00

Source	SS	*df*	MS	*F*	*p*-value	η^2
BETWEEN-SUBJECTS EFFECT						
Condition	7.50	1	7.50	2.50	.153	.238
Error Term Subjects (nested in Condition)	24.00	8	3.00			
WITHIN-SUBJECTS EFFECTS						
Stage of Program	125.00	2	62.50	125.00	< .001	.940
Interaction Stage of Program X Condition	215.00	2	107.50	215.00	< .001	.964
Error Term Interaction Subjects X Stage of Program (nested in Instructional Method	8.00	16	0.50			

SIMPLE MAIN EFFECTS

Source	SS	*df*	MS	*F*	*p*-value	η^2
Condition at Baseline	0	1	0	0	---	.000
Error Term Subjects (nested in Condition)	6.00	8	0.75			
Condition at One Month	160.00	1	160.00	106.67	< .001	.930
Error Term Subjects (nested in Condition)	12.00	8	1.50			
Condition at Three Months	62.50	1	62.50	35.71	< .001	.817
Error Term Subjects (nested in Condition)	14.00	8	1.75			

SPSS Syntax for Simple Main Effects in a Split-Plot Design

Omnibus Model

```
MANOVA
baseline month1 month3 BY cond(1,2)
  /WSFACTORS Time(3)
  / METHOD UNIQUE
  /ERROR WITHIN+RESIDUAL
  /OMEANS TABLES (cond)
  /PMEANS TABLES (cond)
  /POWER T(.05) F(.05)
  / PRINT HOMOGENEITY (BARTLETT BOXM)
      SIGNIF ( AVERF HF GG EFSIZE )
  /NOPRINT PARAM (ESTIM) SIGNIF ( MULT ) .
```

Simple Effects comparing Conditions at each level of Time

```
MANOVA
baseline month1 month3 BY cond(1,2)
  /WSFACTORS Time(3)
  /WSDESIGN MWITHIN Time (1), MWITHIN
Time (2),
       MWITHIN Time (3)
  / METHOD UNIQUE
  /ERROR WITHIN+RESIDUAL
  /POWER T(.05) F(.05)
  / PRINT HOMOGENEITY (BARTLETT BOXM)
SIGNIF ( AVERF HF GG EFSIZE )
  /NOPRINT PARAM (ESTIM) SIGNIF ( MULT ) .
```

Simple Effects comparing Time for each Condition

```
MANOVA
baseline month1 month3 BY cond(1,2)
  /WSFACTORS Time(3)
  / METHOD UNIQUE
  /ERROR WITHIN+RESIDUAL
  /POWER T(.05) F(.05)
  / PRINT HOMOGENEITY (BARTLETT BOXM)
SIGNIF ( AVERF HF GG EFSIZE )
  /NOPRINT PARAM (ESTIM) SIGNIF ( MULT )
  /DESIGN MWITHIN cond (1), MWITH cond (2) .
```

In SPSS 8.0 and higher you can run the **Omnibus Model** through:
Statistics-General Linear Model-Repeated Measures

- Enter **Time** as the **Within-Subjects Factor Name**
- Enter **3** as the **Number of Levels**
- Click **Add** then Click **Define**
- Enter **baseline, month1, month3** in the Upper Box as the **Within-Subjects Variables (1) (2) (3)**, respectively
- Enter **cond** as the **Between-Subjects Factor**
- Click the **Plot** button
- Enter **Time** as the **Horizontal Axis**
- Enter **cond** as **Separate Lines**
- Click **Add** then Click **Continue**
- Click **Options**
- Check the following boxes: **Descriptive statistics, Estimates of effect size, Observed Power, Homogeneity tests**
- Click **Continue** then Click **OK** to RUN THE ANALYSIS

In SPSS 8.0 and higher you can run the **Simple Effects** comparing Conditions at each level of Time through:
Statistics-Compare Means-Means
Enter **baseline, month1, month3** in the **Dependent List**
Enter **cond** in the **Independent List**
Click **Options** and Check the **ANOVA table and eta** box
Click **Continue** then Click **OK** to RUN THE ANALYSIS

SPSS Output from GLM

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.
FACTORB	Pillai's Trace	.969	109.375[a]	2.000	7.000	.000
	Wilks' Lambda	.031	109.375[a]	2.000	7.000	.000
	Hotelling's Trace	31.250	109.375[a]	2.000	7.000	.000
	Roy's Largest Root	31.250	109.375[a]	2.000	7.000	.000
FACTORB* FACTORA	Pillai's Trace	.979	161.875[a]	2.000	7.000	.000
	Wilks' Lambda	.021	161.875[a]	2.000	7.000	.000
	Hotelling's Trace	46.250	161.875[a]	2.000	7.000	.000
	Roy's Largest Root	46.250	161.875[a]	2.000	7.000	.000

a. Exact statistic

Measure: MEASURE_1 **Mauchly's Test of Sphericity**[b]

Within Subject Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse -Geisser	Huynh -Feldt	Lower-Bound
FACTORB	.750	2.014	2	.365	.800	1.00	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom of the averaged tests of significance.
 Corrected test are displyed in the Tests of Within-Subjects Effects table

b. Design: Intercept+FACTORA Within Subject Design: FACTORB

Residual SSCP Matrix

		BASELINE	MONTH1	MONTH3
Sum-of-Squares and Cross-Products	BASELINE	6.000	4.000	8.000
	MONTH1	4.000	12.000	8.000
	MONTH3	8.000	8.000	14.000
Covariance	BASELINE	.750	.500	1.000
	MONTH1	.500	1.500	1.000
	MONTH3	1.000	1.000	1.750
Correlations	BASELINE	1.000	.471	.873
	MONTH1	.471	1.000	.617
	MONTH3	.873	.617	1.000

Calculation of Epsilon

$$\hat{\varepsilon} = \frac{B^2(\bar{s}_{ii} - \bar{s}_{**})^2}{(B - 1)\,(\sum\sum s_{ij}^2 - 2B\sum \bar{s}_i^2 + B^2\bar{s}_{**}^2)},$$

where $s_{ij} = S_{ij}/(N-A)$ is any element of the residual covariance matrix, $\bar{s}_{ii}$=(.75+1.5+1.75)/3=1.33 is the mean of variances, $\bar{s}_{**}$=(.75+.5+1+.5+1.5+1+1+1+1.75)/9=1.0 is the mean of all elements, and $\bar{s}_i$ is the mean of the i^{th} row of the covariance matrix. Thus, $\bar{s}_1$=(.75+.5+1)/3=.75, $\bar{s}_2$=(.5+1.5+1)/3=1, and $\bar{s}_3$=(1+1+1.75)/3=1.25.

$\hat{\varepsilon}$=[(9)(1.33-1.0)^2]/[(2)[(10.375)-((6)(3.125)))+((9)(1))]]

$\hat{\varepsilon}$=0.80

Huynh-Feldt $\tilde{\varepsilon} = \dfrac{N(K-1)\hat{\varepsilon} - 2}{(K-1)(N-J-(K-1)\hat{\varepsilon})}$ =[10(2)(.80)]-2]/[(2)(10-2-((2(.80))=1.094, rounded to 1.0.

$\sum(Y_{ijk} - \bar{Y}_{AB})(Y_{ijk'} - \bar{Y}_{AB'})$

S_{11}= (8-7)(8-7)+(8-7)(8-7)+(7-7)(7-7)
+ (6-7)(6-7)+(6-7)(6-7)
+ (8-7)(8-7)+(7-7)(7-7)+(7-7)(7-7)
+ (7-7)(7-7)+(6-7)(6-7) = 6

S_{22}= (9-8)^2+(8-8)^2+(8-8)^2+(8-8)^2+(7-8)^2
+ (17-16)^2+(18-16)^2+(16-16)^2
+ (14-16)^2+(15-16)^2= 12

S_{33}= (13-12)^2+(14-12)^2+(12-12)^2
+ (10-12)^2+(11-12)^2
+(8-7)^2+(8-7)^2+(7-7)^2+(6-7)^2+(6-7)^2=14

S_{12}= (8-7)(9-8)+(8-7)(8-8)+(7-7)(8-8)
+ (6-7)(8-8)+(6-7)(7-8)
+ (8-7)(17-16)+(7-7)(18-16)
+ (7-7)(16-16)+(7-7)(14-16)
+ (6-7)(15-16) = 4

S_{13}= (8-7)(13-12)+(8-7)(14-12)
+(7-7)(12-12)+(6-7)(10-12)+(6-7)(11-12)
+ (8-7)(8-7)+(7-7)(8-7)+(7-7)(7-7)
+ (7-7)(6-7)+(6-7)(6-7) = 8

S_{23}= (9-8)(13-12)+(8-8)(14-12)
+(8-8)(12-12)+(8-8)(10-12)+(7-8)(11-12)
+ (17-16)(8-7)+(18-16)(8-7)
+ (16-16)(7-7)+(14-16)(6-7)
+ (15-16)(6-7) = 8

SPSS Output from GLM (continued)

Measure: MEASURE_1 **Tests of Within-Subjects Effects**

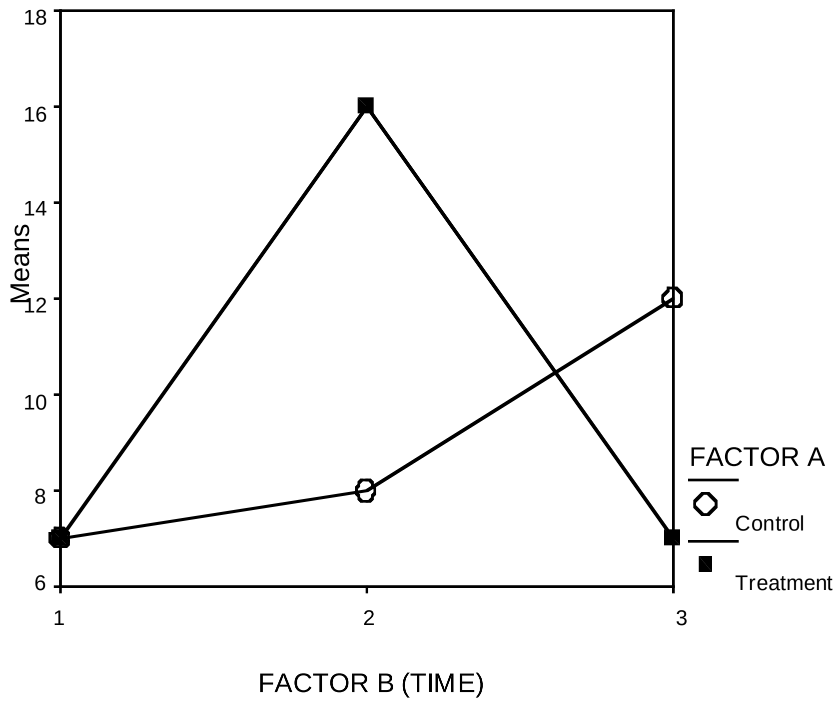
Source		Type III Sum of Squares	df	Mean Square	F	Sig.
FACTORB	Sphericity Assumed	125.000	2	62.500	125.000	.000
	Greenhouse-Geisser	125.000	1.600	78.125	125.000	.000
	Huynh-Feldt	125.000	2.000	62.500	125.000	.000
	Lower-Bound	125.000	1.000	125.000	125.000	.000
FACTORB* FACTORA	Sphericity Assumed	215.000	2	107.500	215.000	.000
	Greenhouse-Geisser	215.000	1.600	134.375	215.000	.000
	Huynh-Feldt	215.000	2.000	107.500	215.000	.000
	Lower-Bound	215.000	1.000	215.000	215.000	.000
ERROR(FACTORB)	Sphericity Assumed	8.000	16	.500		
	Greenhouse-Geisser	8.000	12.800	.625		
	Huynh-Feldt	8.000	16.000	.500		
	Lower-Bound	8.000	8.000	1.000		

Measure: MEASURE_1 **Tests of Within-Subjects Contrasts**

Effect		Type III Sum of Squares	df	Mean Square	F	Sig.
FACTORB	Linear	31.250	1	31.250	125.000	.000
	Quadratic	93.750	1	93.750	125.000	.000
FACTORB* FACTORA	Linear	31.250	1	31.250	125.000	.000
	Quadratic	183.750	1	183.750	245.000	.000
ERROR(FACTORB)	Linear	2.000	8	.250		
	Quadratic	6.000	8	.750		

Measure: MEASURE_1
Transformed Variable: Average **Tests of Between-Subjects Effects**

Effect	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	2707.500	1	2707.500	902.500	.000
FACTORA	7.500	1	7.500	2.500	.153
ERROR	24.000	8	3.000		



SPSS Output from Compare Means

Report

FACTORA		BASELINE	MONTH1	MONTH3	PMEAN
Control	Mean	7.0000	8.0000	12.0000	9.0000
	N	5	5	5	5
	Std. Deviation	1.0000	.7071	1.5811	1.0000
Treatment	Mean	7.0000	16.0000	7.0000	10.0000
	N	5	5	5	5
	Std. Deviation	.7071	1.5811	1.0000	1.0000
Total	Mean	7.0000	12.0000	9.5000	9.5000
	N	10	10	10	10
	Std. Deviation	.8165	4.3716	2.9155	1.0801

ANOVA Table

		Type III Sum of Squares	df	Mean Square	F	Sig.	Eta	Eta Squared
BASELINE * FACTORA	Between Groups	.000	1	.000	.000	1.000	.000	.000
	Within Groups	6.000	8	.750				
	Total	6.000	9					
MONTH1 * FACTORA	Between Groups	215.000	1	160.000	106.667	.000	.964	.930
	Within Groups	215.000	8	1.500				
	Total	215.000	9					
MONTH3 * FACTORA	Between Groups	62.500	1	62.500	35.714	.000	.904	.817
	Within Groups	14.000	8	1.750				
	Total	76.500	9					
PMEAN * FACTORA	Between Groups	2.500	1	2.500	2.500	.153	.488	.238
	Within Groups	8.000	8	1.000				
	Total	10.500	9					

Note: That values for PMEAN are identical to the Between-Subjects from the Split-Plot ANOVA

General Linear Model Approach

$Y_{ijk} = \mu_*$	$+ \alpha_j$	$+ \pi_{i(j)}$	$+ \beta_k$	$+ \alpha\beta_{jk}$	$+ \beta\pi_{i(j)}$
$Y_{ijk} = \bar{Y}_*$	$+ a_j$	$+ p_{i(j)}$	$+ b_k$	$+ ab_{jk}$	$+ bp_{i(j)}$
8 = 9.5	-0.5	+1	-2.5	+0.5	+0
8 = 9.5	-0.5	+1	-2.5	+0.5	+0
7 = 9.5	-0.5	+0	-2.5	+0.5	+0
6 = 9.5	-0.5	-1	-2.5	+0.5	+0
6 = 9.5	-0.5	-1	-2.5	+0.5	+0
8 = 9.5	+0.5	+1	-2.5	-0.5	+0
7 = 9.5	+0.5	+1	-2.5	-0.5	-1
7 = 9.5	+0.5	+0	-2.5	-0.5	+0
7 = 9.5	+0.5	-1	-2.5	-0.5	+1
6 = 9.5	+0.5	-1	-2.5	-0.5	+0
9 = 9.5	-0.5	+1	+2.5	-3.5	+0
8 = 9.5	-0.5	+1	+2.5	-3.5	-1
8 = 9.5	-0.5	+0	+2.5	-3.5	+0
8 = 9.5	-0.5	-1	+2.5	-3.5	+1
7 = 9.5	-0.5	-1	+2.5	-3.5	+0
17 = 9.5	+0.5	+1	+2.5	+3.5	+0
18 = 9.5	+0.5	+1	+2.5	+3.5	+1
16 = 9.5	+0.5	+0	+2.5	+3.5	+0
14 = 9.5	+0.5	-1	+2.5	+3.5	-1
15 = 9.5	+0.5	-1	+2.5	+3.5	+0
13 = 9.5	-0.5	+1	+ 0	+3.0	+0
14 = 9.5	-0.5	+1	+ 0	+3.0	+1
12 = 9.5	-0.5	+0	+ 0	+3.0	+0
10 = 9.5	-0.5	-1	+ 0	+3.0	-1
11 = 9.5	-0.5	-1	+ 0	+3.0	+0
8 = 9.5	+0.5	+1	+ 0	-3.0	+0
8 = 9.5	+0.5	+1	+ 0	-3.0	+0
7 = 9.5	+0.5	+0	+ 0	-3.0	+0
6 = 9.5	+0.5	-1	+ 0	-3.0	+0
6 = 9.5	+0.5	-1	+ 0	-3.0	+0
$SS_Y =$ $\Sigma(Y_{ij} - \mu_{**})^2$	$\Sigma\alpha^2$ + $= \Sigma(\mu_{j*} - \mu_{**})^2$	$\Sigma\pi^2$ + $= \Sigma(\mu_{i*} - \mu_{j*})^2$	$\Sigma\beta^2$ + $= \Sigma(\mu_{*k} - \mu_{**})^2$	$\Sigma\alpha\beta^2$ + $= \Sigma(\mu_{jk} - \mu_{j*} - \mu_{*k} + \mu_{**})^2$	$\Sigma\beta\pi^2$ $= \Sigma(Y_{ij} - \mu_{j*} - \mu_{*k} + \mu_{j*})^2$
$\Sigma(Y_{ij} - \bar{Y}_{**})^2$	$= \Sigma(\bar{Y}_{j*} - \bar{Y}_{**})^2$	$= \Sigma(\bar{Y}_{i*} - \bar{Y}_{j*})^2$	$= \Sigma(\bar{Y}_{*k} - \bar{Y}_{**})^2$	$= \Sigma(\bar{Y}_{jk} - \bar{Y}_{j*} - \bar{Y}_{*k} + \bar{Y}_{**})^2$	$= \Sigma(Y_{ij} - \bar{Y}_{i*} - \bar{Y}_{*k} + \bar{Y}_{j*})^2$
$SS_Y =$ 379.5	$= (SS_A$ **=7.5**)	$= (SS_P =$ **24**)	$= (SS_B =$ **125**)	$= (SS_{AB} =$ **215**)	$= (SS_{BP} =$ **= 8**)

The Multivariate Approach to Repeated Measures.

A matrix of (K-1) difference scores are created and submitted to a MANOVA. The difference matrix can be defined many ways.

For example, pairwise differences may be calculated. In the present example, d_1 = (Baseline- Month 1) and d_2 = Month 1 - Month 3). In matrix notation, define **Y** as the $N \times K$ matrix of the Baseline, Month 1 and Month 3 scores. The difference matrix **C** = {1 -1 0} {0 1 -1}. Therefore, **Y** times the transpose of **C** will be the scores transformed by the difference matrix which will be referred to as $\mathbf{D} = \mathbf{Y}\mathbf{C}'$.

The columns of **D**, d1 and d2, may be computed with the following SPSS Syntax..

```
COMPUTE d1 = (baseline month1) .
COMPUTE d2 = (month1 - month3) .
EXECUTE.
```

There are important concepts to note. First of all if the null hypothesis for the Repeated Measures main effect were true, then all pairwise difference should be equal to 0. The basis for this goes all the way back to the dependent t-test, where differences between 2 dependent measures are computed. Then a t-statistic is computed to test whether the difference scores equals 0. In this multivariate context more than one set of difference scores is tested, but they are all assumed to equal zero under the null hypothesis. The multivariate test for Intercept from a one-way MANOVA performed on difference scores is equivalent to the multivariate test of the Repeated Measures main effect.

Recall that interactions have been discussed as differences in differences. In this context the Treatment effect (the difference between Treatment and Control group means) was not constant across the phases of the study. The Treatment effect was 0 = (7-7) at Baseline, 8 = (16-8) at Month 1, and -5 = (7-12) at Month 3. This interaction will be reflected in the fact that the transformed difference scores **T** are not the same for the 2 groups (Treatment and Control). Thus, the multivariate test for the Group Effect from a one-way MANOVA performed on difference scores is equivalent to the multivariate test of the Repeated Measures by Between-Subjects Group interaction.

It has been noted that the hypothesis degrees-of-freedom (df_h) for the one-way MANOVA F test is equal to $p(A - 1)$, where p = the number of variables. In this case, the df_h equals 2 times the number of groups minus one [$df_h = p(A - 1) = 2$]. Note that the number of variables equals the number of Repeated Measures minus one which is the degrees of freedom for the Repeated Measures effect. Thus, the df_h for the one-way MANOVA performed on difference scores is equal to $(A\text{-}1)(B\text{-}1)$, which is the degrees of freedom for the Interaction from the Split-Plot ANOVA.

It should be noted that any kind of difference matrix will lead to the same results.
For illustrating other important concepts that distinguish the multivariate and univariate approach to Repeated Measure design another type of difference matrix will be introduced. This matrix is know as an orthonormal trend matrix. "Ortho" comes from the fact that orthogonal polynomial trend coefficients are the basis for this matrix. With B = 3 Repeated Measures, B -1 = 2 trends can be assess, Linear and Quadratic. In the present example you may already note that the trend for the Control Group looks rather linear with means becomes larger over time. The trend for the Treatment Group is more quadratic with the mean going up at Month 1 but coming back down to the Baseline level at Month 3.
The linear polynomial for 3 measures has coefficients of -1 0 +1. Note that if these were plotted they would form a line.
The quadratic polynomial has coefficients of +1 -2 +1. Note that if these coefficients were plotted that would form a "U-shape" that is similar to plotting squared values.
The "normal" part of orthonormal is that these coefficients are divided by their standard deviation so that they will not change the overall variance of Y.
Thus, the linear coefficients are divided by 1.41 = $\sqrt{(-1^2)+(0^2)+(+1^2)}$.
The quadratic coefficients are divided by 2.45 = $\sqrt{(+1^2)+(-2^2)+(+1^2)}$.

Thus, this orthonormal transformation matrix for trends is **R** = { -0.707 0 +0.707
Therefore, **Y** times the transpose of **R** will be the scores +0.408 -0.816 +0.408}.
transformed by the difference matrix which will be referred to as **T** = **YR′**.
The columns of **T**, t1 and t2, may be computed with the following SPSS Syntax..

```
COMPUTE t1 = ((-1*baseline)+(0*month1)+(1*month3))/(SQRT(2)).
COMPUTE t2 = ((1*baseline)+(-2*month1)+(1*month3))/(SQRT(6)).
EXECUTE.
```

Note that for the Main Effect of Factor B, the Factor A*Factor B Interaction, and the Error term the Sums of Squares for the Linear and Quadratic trend add up to the Sums of Squares from the omnibus Source Table. What has happened is that the Transformation matrix R has transformed the **Y** matrix with *B* Repeated Measures into a matrix, **T**, with *B*-1 variables that represent the orthogonal (and additive) contribution of the Linear and Quadratic trends underlying the data. For example, the Error SS for the Linear trend equals 2 and the Error SS for the Quadratic trend equals 6. These sum to 8 the Error term used in the Split-Plot ANOVA. One property that distinguishes the multivariate approach from the univariate approach to Repeated Measures Designs is that the univariate approach only uses the trace (i.e., variance terms on the diagonal) of the Error and Hypothesis matrices. The multivariate tests incorporate the off-diagonal covariance terms. Thus, when variables are correlated the multivariate approach gains power. Also, in the Repeated measures context, because the univariate approach does not incorporate the off diagonal covariance terms, there is an implicit assumptions that they will have no effect on the test. The only way the covariance terms will not affect the F test is if all of the off-diagonal covariance terms are equal. This is the assumption of sphericity. By incorporating the covariance terms, the multivariate tests do not have an assumption of sphericity.

	cond	baseline	month1	month3	pmean	d1	d2	t1	t2
1	Control	8	9	13	10	-1	-4	3.536	1.225
2	Control	8	8	14	10	0	-6	4.243	2.449
3	Control	7	8	12	9	-1	-4	3.536	1.225
4	Control	6	8	10	8	-2	-2	2.828	.000
5	Control	6	7	11	8	-1	-4	3.536	1.225
6	Treat	8	17	8	11	-9	9	.000	-7.348
7	Treat	7	18	8	11	-11	10	.707	-8.573
8	Treat	7	16	7	10	-9	9	.000	-7.348
9	Treat	7	14	6	9	-7	8	-.707	-6.124
10	Treat	6	15	6	9	-9	9	.000	-7.348

All Measures have Standard Deviationof $s = 1$.
(Equivalent to Correlation Matrix).

$K = 4$			
1.0000	0.5000	0.5000	0.5000
	1.0000	0.5000	0.5000
		1.0000	0.5000
	ε = 1.0000		1.0000
1.0000	0.7000	0.5000	0.3000
	1.0000	0.7000	0.5000
		1.0000	0.7000
	ε = 0.7511		1.0000
1.0000	0.8000	0.5000	0.2000
	1.0000	0.8000	0.5000
		1.0000	0.8000
	ε = 0.5333		1.0000

All Measures have Standard Deviationof $s = 3$.

$K = 4$			
9.0000	1.5000	1.5000	1.5000
	9.0000	1.5000	1.5000
		9.0000	1.5000
	ε = 1.0000		9.0000
9.0000	2.1000	1.5000	0.9000
	9.0000	2.1000	1.5000
		9.0000	2.1000
	ε = 0.7511		9.0000
9.0000	2.4000	1.5000	0.6000
	9.0000	2.4000	1.5000
		9.0000	2.4000
	ε = 0.5333		9.0000

All Measures have Standard Deviation of $s = 1$.
(Equivalent to Correlation Matrix).

$K = 4$			
1.0000	0.5000	0.5000	0.5000
	1.0000	0.5000	0.5000
		1.0000	0.5000
	ε = 1.0000		1.0000
1.0000	0.7000	0.5000	0.3000
	1.0000	0.7000	0.5000
		1.0000	0.7000
	ε = 0.7511		1.0000
1.0000	0.8000	0.5000	0.2000
	1.0000	0.8000	0.5000
		1.0000	0.8000
	ε = 0.5333		1.0000

One Measures has Standard Deviationof $s = 3$.
All other measures has Standard Deviation of $s = 1$.

$K = 4$			
1.0000	0.5000	**1.5000**	0.5000
	1.0000	**1.5000**	0.5000
		9.0000	**1.5000**
	ε = 0.4706		1.0000
1.0000	0.7000	**1.5000**	0.3000
	1.0000	**2.1000**	0.5000
		9.0000	**2.1000**
	ε = 0.4779		1.0000
9.0000	**2.4000**	**1.5000**	**0.6000**
	1.0000	0.8000	0.5000
		1.0000	0.8000
	ε = 0.3772		1.0000

$$\varepsilon = \frac{K^2(\bar{\sigma}_{ii} - \bar{\sigma}_{**})^2}{(K-1)(\sum\sum\bar{\sigma}_{ij}^2 - 2K\sum\bar{\sigma}_{ij}^2 + K^2\bar{\sigma}_{**}^2)}$$

where σ_{ij} is any element of the population covariance matrix, $\bar{\sigma}_{ii}$ is the mean of the variances, $\bar{\sigma}_{**}$ is the mean of all elements in the population covariance matrix, and $\bar{\sigma}_i$ is the mean of the i^{th} row (or column) of the covariance matrix.

$$\varepsilon = \frac{[\mathrm{tr}(\mathbf{C}_K \Sigma \mathbf{C}_K')]^2}{(K-1)[\mathrm{tr}(\mathbf{C}_K \Sigma \mathbf{C}_K')^2]}$$

where $\mathbf{C}_K$ is $(K\text{-}1)$ x K transformation matrix transforming the K repeated measures into $(K\text{-}1)$ difference variables of the general form:

$$\mathbf{C}_K = \begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{matrix}$$