This is an Exploratory Factor Analysis of data related to a unidimensional Questionnaire.

	q 1	q 2	q 3	q 4	q 5	q 6	q 7	total	fac1_1
1	2	1	1	1	1	2	1	9	-1.6620
2	2	1	1	1	1	2	1	9	-1.6620
3	2	1	2	1	2	2	2	12	-.9566
4	4	1	2	1	2	2	2	14	-.6644
5	2	1	2	1	2	2	2	12	-.9566
6	1	1	2	1	2	1	2	10	-1.3025
7	3	2	2	1	2	2	2	14	-.5723
8	3	1	2	1	2	2	2	13	-.8105
9	4	1	2	1	2	3	3	16	-.2470
10	2	1	2	1	2	2	2	12	-.9566
11	2	1	2	1	2	2	2	12	-.9566
12	2	2	2	1	2	2	2	13	-.7184
13	2	1	2	2	2	3	2	14	-.5861
14	2	1	2	2	2	3	3	15	-.3685
15	2	1	3	1	2	2	2	13	-.7492
16	3	2	3	1	3	4	4	20	.7503
17	4	1	3	2	2	3	3	18	.1311
18	2	2	3	2	3	2	2	16	-.0600
19	2	2	3	2	3	3	3	18	.3575
20	3	2	3	1	3	4	4	20	.7503
21	4	3	3	3	3	4	3	23	1.2583
22	4	3	3	3	3	4	3	23	1.2583
23	4	3	3	3	3	3	4	23	1.2761
24	3	2	4	3	3	3	3	21	.8816
25	3	2	4	3	3	4	3	22	1.0815
26	4	3	3	3	3	4	4	24	1.4759
27	2	2	3	4	3	3	3	20	.6988
28	4	2	4	3	3	3	3	22	1.0277
28	2	2	4	3	3	2	3	19	.5357
30	4	3	4	3	4	4	3	25	1.7461

datafile: onefacteg.sav

q1 through **q7** are responses to 7 items of a Likert-type questionnaire with 4 response options.

SPSS output from the Factor Analysis module.

Correlation Matrix

		q1	q2	q3	q4	q5	q6	q7
Correlation	q1	1.000	.536	.409	.381	.436	.643	.553
	q2	.536	1.000	.633	.699	.806	.667	.644
	q3	.409	.633	1.000	.731	.882	.564	.678
	q4	.381	.699	.731	1.000	.717	.569	.540
	q5	.436	.806	.882	.717	1.000	.671	.762
	q6	.643	.667	.564	.569	.671	1.000	.771
	q7	.553	.644	.678	.540	.762	.771	1.000

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	q4	.381	.699	.731	1.000	.717	.569	.540
	q5	.436	.806	.882	.717	1.000	.671	.762
	q6	.643	.667	.564	.569	.671	1.000	.771
	q7	.553	.644	.678	.540	.762	.771	1.000

Communalities

	Initial	Extraction
q1	1.000	.436
q2	1.000	.745
q3	1.000	.726
q4	1.000	.644
q5	1.000	.846
q6	1.000	.703
q7	1.000	.732

Extraction Method: Principal Components Analysis.

I chose to analyze the Correlation matrix, instead of the covariance matrix, and chose a **Principal Components Analysis** (PCA), instead of Principal Axis Factoring, in order to demonstrate some statistical properties.

One Factor was extracted based on the eigenvalues in context with theoretical expectations.

$\Sigma\lambda = \Sigma h^2_{initial} = 7$ $\Sigma h^2_{ext} = 4.832$ (4.832/7.00)x100=69.035% of the variance among the 7 variables is explained by a one-factor solution.

The **Initial** (or prior) communalities are the diagonals of the matrix analyzed. In a PCA of the Correlation matrix (**R**), all the diagonal entries (i.e., initial communalities) are 1.

The **Extraction** (or posterior) communalities (h^2) are the proportion of each variable explained by the factor structure. Thus, it is an R^2 for how well the factor structure explains each variable.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	4.832	69.035	69.035	4.832	69.035	69.035
2	.838	11.972	81.007			
3	.453	6.478	87.485			
4	.346	4.947	92.432			
5	.297	4.244	96.676			
6	.174	2.483	99.159			
7	5.887E-02	.841	100.000			

$\Sigma\lambda = 7$ $\Sigma\lambda = 4.832$ (4.832/7) = .69035

Extraction Method: Principal Components Analysis.

a. When components are correlated, sum of squared loadings cannot be added to obtain a total variance.

Because 1s are used in the diagonal of a PCA, the trace of the matrix equals the number of variables (7 in this case). Since the Sum of the Eigenvalues equals the Trace of the matrix, the Sum of the Eigenvalues equals the number of variables.

Component Matrix (U)

	Component 1
q1	**.660**
q2	**.863**
q3	**.852**
q4	**.803**
q5	**.920**
q6	**.839**
q7	**.856**

$h^2 = .660^2 = \mathbf{.436}$
$h^2 = .863^2 = \mathbf{.745}$
$h^2 = .852^2 = \mathbf{.726}$
$h^2 = .803^2 = \mathbf{.644}$
$h^2 = .920^2 = \mathbf{.846}$
$h^2 = .839^2 = \mathbf{.703}$
$h^2 = .856^2 = \mathbf{.732}$

Eigenvalue $= \Sigma h^2 =$ **4.832**

Correlation Matrix

		fac1_1	total
Correlation N = 30	q1	**.660**	**.696**
	q2	**.863**	**.854**
	q3	**.852**	**.841**
	q4	**.803**	**.807**
	q5	**.920**	**.899**
	q6	**.839**	**.846**
	q7	**.856**	**.850**

Note: . All correlations are significant at $p < .001$. The correlation between the Total scores and the Factor Scores is r = .998.

Component Score Coefficient Matrix (B)

	Component 1
q1	**.137**
q2	**.179**
q3	**.176**
q4	**.166**
q5	**.190**
q6	**.174**
q7	**.177**

These are weights applied to the standardized variables in order to create Factor Scores. For example, to create Factor scores for the factor (fac1_1 in the data set above), you would convert the variables to z-scores and compute:

fac1_1 = .137(z1) +.179(z2) +.176(z3) +.166(z4) +.190(z5) +.174(z6) +.177(z7)

In matrix notation, the matrix of Factor score, $\mathbf{M}_{30,1,} = \mathbf{Z}_{30,7}\mathbf{B}_{7,1}$

SPSS software computes these automatically and will save them if requested.

Scoring Coefficients as a Regression Model

The R^2 of 1.00 indicates that the Factor Scores are perfectly described by a linear combination of the variables q1 - q7.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	1.000 a	**1.000**	1.000	1.20947E-08

a Predictors: (Constant), q1, q2, q3, q4, q5, q6, q7

Coefficients

	Unstandardized Coefficients		Standardized Coefficients			Correlations			Collinearity Statistics	
	B	Std. Error	Beta	t	Sig.	Zero-order	Partial	Part	Tol.	VIF
1 **(Constant)**	-3.468									
q1	.146	.000	**.137**	-	-	**.660**	1.000	.098	.513	1.949
q2	.238	.000	**.179**	-	-	**.863**	1.000	.085	.229	4.366
q3	.207	.000	**.176**	-	-	**.852**	1.000	.070	.159	6.303
q4	.171	.000	**.166**	-	-	**.803**	1.000	.098	.351	2.849
q5	.280	.000	**.190**	-	-	**.920**	1.000	.059	.098	10.242
q6	.200	.000	**.174**	-	-	**.839**	1.000	.095	.299	3.346
q7	.218	.000	**.177**	-	-	**.856**	1.000	.094	.282	3.544

a Dependent Variable: fac1_1

Eigenvalue $= \Sigma r^2 =$ **4.832**

From the Fundamental Theorem of Principal Components, the original correlation matrix (**R**) can be reproduced by $\mathbf{R}_{(r)} = \mathbf{U}_{p,f}\mathbf{U}'_{f,p}$.

Correlation Matrix

		q1	q2	q3	q4	q5	q6	q7
Reproduced Correlation	**q1**	.436[b]	.570	.563	.530	.607	.554	.565
	q2	.570	.745[b]	.736	.693	.794	.724	.738
	q3	.563	.736	.726[b]	.684	.784	.715	.729
	q4	.530	.693	.684	.644[b]	.738	.673	.687
	q5	.607	.794	.784	.738	.846[b]	.771	.787
	q6	.554	.724	.715	.673	.771	.703[b]	.717
	q7	.565	.738	.729	.687	.787	.717	.732[b]
Residual[a]	**q1**		-.034	-.154	-.149	-.171	.089	-.012
	q2	-.034		-.103	.007	.012	-.056	-.094
	q3	-.154	-.103		.047	.098	-.151	-.051
	q4	-.149	.007	.047		-.021	-.104	-.147
	q5	-.171	.012	.098	-.021		-.100	-.025
	q6	.089	-.056	-.151	-.104	-.100		.054
	q7	-.012	-.094	-.051	-.147	-.025	.054	

a Residuals are computed between observed and reproduced correlations. There are 14 (66.0%) nonredundant residuals with absolute values > 0.05.
b Reproduced communalities

Reproduced Correlation Matrix: $\mathbf{R}_{(r)7,7} = \mathbf{U}_{7,1}\mathbf{U}'_{1,7}$

R E L I A B I L I T Y A N A L Y S I S - S C A L E (A L P H A)

	Mean	Std Dev	Cases
1. Q1	2.7667	.9353	30.0
2. Q2	1.7000	.7497	30.0
3. Q3	2.6333	.8503	30.0
4. Q4	1.8667	.9732	30.0
5. Q5	2.4333	.6789	30.0
6. Q6	2.7333	.8683	30.0
7. Q7	2.6000	.8137	30.0

Statistics for	Mean	Variance	Std Dev	N of Variables
SCALE	16.7333	23.3057	4.8276	7

Item-total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Alpha if Item Deleted
Q1	13.9667	17.8954	.5732	.9245
Q2	15.0333	17.6885	.8016	.9000
Q3	14.1000	17.1276	.7751	.9015
Q4	14.8667	16.6713	.7156	.9094
Q5	14.3000	17.8724	.8662	.8963
Q6	14.0000	16.9655	.7809	.9008
Q7	14.1333	17.2920	.7908	.9001

Reliability Coefficients

N of Cases = 30.0 N of Items = 7

Alpha = **.9172**