This is an Exploratory Factor Analysis of data related to Anxiety and Self-Efficacy in Statistics.

	X1	X2	X3	X4	X5	X6	X7	X8	fac1_obl	fac2_obl	fac1_paf	fac2_paf
1	2	1	1	1	2	2	2	1	-.92456	-1.56799	-.692200	-1.44408
2	2	1	1	1	2	2	2	2	-.56600	-1.52358	-.55760	-1.35508
3	2	1	2	1	2	3	2	1	-.51311	-1.09420	-.40076	-.88593
4	4	1	2	1	2	3	3	1	-.32721	-.66577	-.19114	-.79478
5	2	1	2	1	2	2	2	1	-.80651	-1.08562	-.70230	-.78456
6	1	1	2	1	3	4	4	2	1.28814	-1.19041	1.29633	-1.06326
7	3	2	2	1	2	3	3	1	-.27747	-.44454	-.18204	-.54531
8	3	1	2	1	3	2	2	2	-.10361	-.78818	.02360	-.72289
9	4	1	2	1	3	3	3	2	.44789	-.57532	.57649	-.78644
10	2	1	2	1	3	4	4	2	1.21595	-.98343	1.25459	-1.00994
11	2	1	2	1	3	4	3	3	1.24422	-.95346	1.09609	-.90545
12	2	2	2	1	3	4	3	3	1.22177	-.52524	1.06346	-.60266
13	2	1	2	2	3	3	4	3	1.23264	-.62091	1.08537	-.62305
14	2	1	2	2	3	3	3	2	.54378	-.67977	.65768	-.69656
15	2	1	3	1	3	4	3	2	1.00371	-.51551	.95140	-.33493
16	3	2	3	1	3	4	4	3	1.59793	.17857	1.30472	.09470
17	4	1	3	2	3	3	3	2	.51746	.21657	.56411	.06960
18	2	2	3	2	3	3	3	2	.63939	.23082	.61495	.26575
19	2	2	3	2	3	2	3	2	.34599	.23940	.31341	.36713
20	3	2	3	1	4	4	3	3	1.68416	.21016	1.64466	.02953
21	4	3	3	3	1	2	1	1	-1.72151	1.22573	-1.79184	1.07636
22	4	3	3	3	1	2	1	1	-1.72151	1.22573	-1.79184	1.07636
23	4	3	3	3	2	2	2	1	-.97470	1.28622	-.86571	.98022
24	3	2	4	3	2	2	2	1	-.76200	1.13338	-.80143	1.28363
25	3	2	4	3	2	2	2	1	-.76200	1.13338	-.80143	1.28363
26	4	3	3	3	2	1	2	1	-1.26809	1.29480	-1.16724	1.08160
27	2	2	3	4	2	2	2	2	-.49778	.79795	-.61728	.85630
28	4	2	4	3	2	2	2	1	-.83419	1.34037	-.84317	1.33695
28	2	2	4	3	2	3	3	1	-.06613	.93226	-.16506	1.11344
30	4	3	4	3	2	2	2	1	-.85664	1.76859	-.87580	1.63974

datafile: twofacexcor.sav

SPSS output from the Factor Analysis module.

Correlation Matrix

		X1	X2	X3	X4	X5	X6	X7	X8
Correlation	X1	1.000	.536	.409	.381	-.378	-.419	-.444	-.398
	X2	.536	1.000	.633	.699	-.481	-.445	-.486	-.350
	X3	.409	.633	1.000	.731	-.193	-.230	-.219	-.287
	X4	.381	.699	.731	1.000	-.536	-.615	-.505	-.435
	X5	-.378	-.481	-.193	-.536	1.000	.671	.762	.806
	X6	-.419	-.445	-.230	-.615	.671	1.000	.771	.667
	X7	-.444	-.486	-.219	-.505	.762	.771	1.000	.644
	X8	-.398	-.350	-.287	-.435	.806	.667	.644	1.000

X1 is the **Speilberger Trait Anxiety Scale**.

X2 is the **Speilberger State Anxiety Scale.**

X3 is the **Math Anxiety Rating Scale**.

X4 is the **Statistics Anxiety Rating Scale**.

X5 is a measure of **General Self-Efficacy**.

X6 is a measure of **Academic Self-Efficacy**.

X7 is a measure of **Self-Efficacy in the use of Computers**.

X8 is a measure of **Self-Efficacy in Mathematics Related Content**.

Communalities

	Initial	Extraction
X1	1.000	.441
X2	1.000	.763
X3	1.000	.848
X4	1.000	.790
X5	1.000	.837
X6	1.000	.766
X7	1.000	.800
X8	1.000	.748

Extraction Method: Principal Components Analysis.

I chose analyze the Correlation matrix, instead of the covariance matrix, and chose a **Principal Components Analysis**, instead of Principal Axis Factoring, in order to demonstrate some statistical properties.

Two Factors were extracted based on the eigenvalues in context with theoretical expectations.

$\Sigma\lambda = \Sigma h^2_{initial} = 8$ $\Sigma h^2_{ext} = 5.993$ (5.993/8)x100=74.91% of the variance among the 8 variables is explained by a two-factor solution.

The **Initial** (or prior) communalities are the diagonals of the matrix analyzed. In a PCA of the Correlation matrix (**R**), all the diagonal entries (i.e., initial communalities) are 1.

The **Extraction** (or posterior) communalities (h^2) are the proportion of each variable explained by the factor structure. Thus, it is an R^2 for how well the factor structure explains each variable.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	4.582	57.269	57.269	4.582	57.269	57.269	4.011
2	1.411	17.639	74.908	1.411	17.639	74.908	3.395
3	.684	8.545	83.453				
4	.470	5.873	89.326				
5	.375	4.692	94.018				
6	.230	2.873	96.891				
7	.188	2.353	99.245				
8	.060	.755	100.000				

Because 1s are used in the diagonal of a PCA, the trace of the matrix equals the number of variables (8 in this case). Since the Sum of the Eigenvalues equals the Trace of the matrix. the Sum of the Eigenvalues equals the number of variables.

$\Sigma\lambda = 8$ $\Sigma\lambda = 5.993$ (5.993/8) = .74908

Extraction Method: Principal Components Analysis.

a. When components are correlated, sum of squared loadings cannot be added to obtain a total variance.

Correlation Matrix

		X1	X2	X3	X4	X5	X6	X7	X8
Reproduced Correlation	X1	.441[b]	.565	.507	.585	-.446	-.461	-.453	-.418
	X2	.565	.763[b]	.752	.773	-.445	-.482	-.464	-.415
	X3	.507	.752	.848[b]	.735	-.191	-.254	-.223	-.173
	X4	.585	.773	.735	.790[b]	-.514	-.545	-.530	-.481
	X5	-.446	-.445	-.191	-.514	.837[b]	.797	.817	.791
	X6	-.461	-.482	-.254	-.545	.797	.766[b]	.782	.753
	X7	-.453	-.464	-.223	-.530	.817	.782	.800[b]	.772
	X8	-.418	-.415	-.173	-.481	.791	.753	.772	.748[b]
Residual[a]	X1		-.029	-.098	-.204	.067	.042	.009	.019
	X2	-.029		-.119	-.074	-.036	.037	-.023	.065
	X3	-.098	-.119		-.004	-.002	.023	.003	-.113
	X4	-.204	-.074	-.004		-.021	-.069	.025	.046
	X5	.067	-.036	-.002	-.021		-.127	-.056	.015
	X6	.042	.037	.023	-.069	-.127		-.011	-.086
	X7	.009	-.023	.003	.025	-.056	-.011		-.128
	X8	.019	.065	-.113	.046	.015	-.086	-.128	

b Reproduced communalities

Reproduced Correlation Matrix: $\mathbf{R}_{(r)8,8} = \mathbf{S}_{8,2}\mathbf{P}'_{2,8}$

h^2=[(.838)(.873)] +[(-.076)(-.459)] =**.766**

Component Matrix[a] (U)

	Component 1	Component 2
X1	-.636	.193
X2	-.754	.440
X3	-.580	.715
X4	-.807	.374
X5	.823	.400
X6	.819	.309
X7	.821	.354
X8	.774	.385

Σu^2 = 4.582 + 1.411 = 5.993

Pattern Matrix[b] (P)

	Component 1	Component 2
X1	-.265	**.500**
X2	-.127	**.808**
X3	.237	**1.005**
X4	-.223	**.765**
X5	**.921**	.015
X6	**.838**	-.076
X7	**.880**	-.031
X8	**.875**	.023

Structure Matrix (S)

	Component 1	Component 2	ps 1	ps 2	Σps h^2
X1	-.494	.621	.131	.310	.441
X2	-.497	.866	.063	.700	.763
X3	-.222	.896	-.052	.900	.848
X4	-.572	.867	.127	.663	.790
X5	.915	-.406	.843	-.007	.837
X6	.873	-.459	.731	.035	.766
X7	.894	-.433	.787	.013	.800
X8	.865	-.377	.757	-.009	.748

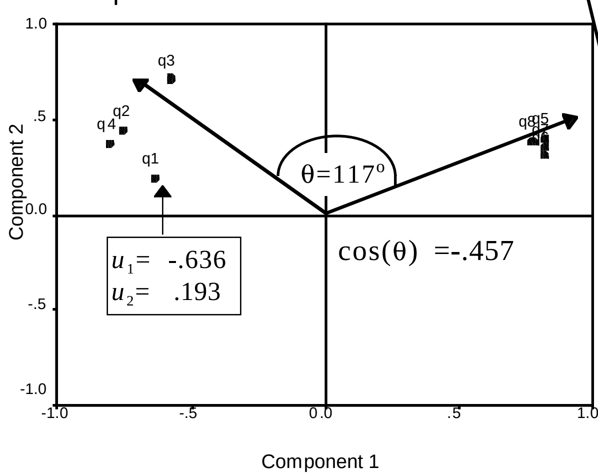
Σps = 3.387 + 2.606 = 5.993 = Σh^2

a. Extraction Method: Principal Components Analysis.
b. Rotation Method: Oblimin with Kaiser Normalization.
a. 2 components extracted
b. Rotation converged in 6 iterations.

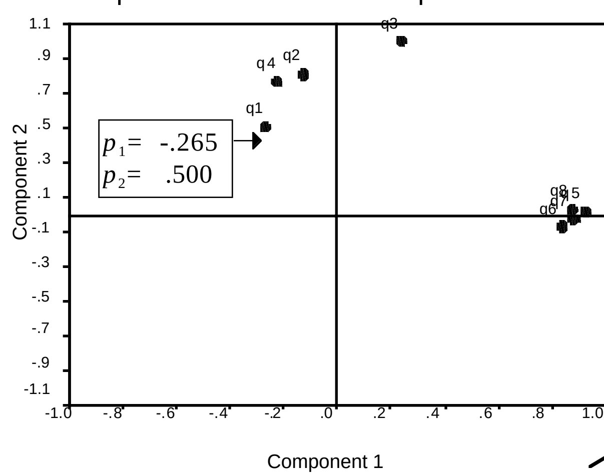
Component Correlation Matrix

Component	1	2
1	1.000	-.457
2	-.457	1.000

Component Plot



Component Plot in Rotated Space



Example of Pattern Matrix as a Regression Equation

$X_6 = p_{61}F_1 + p_{62}F_2$

$X_6 = .838F_1 - .076F_2$

. . . .

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.875a	.766	.749	.440

a Predictors: (Constant), Anxiety (F1), Self-Eff (F2)

Coefficients

	Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B		Correlations			Collinearity Statistics	
	B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound	Zero-order	Partial	Part	Tol.	VIF
1 **(Constant)**	2.733	.079		34.407	.000	2.570	2.896					
Anxiety (F1)	.728	.091	**.838**	8.011	.000	.541	.914	**.873**	.839	.745	.791	1.264
Self-Eff (F2)	-.066	.091	**-.076**	-.726	.474	-.252	.120	**-.459**	-.138	-.068	.791	1.264

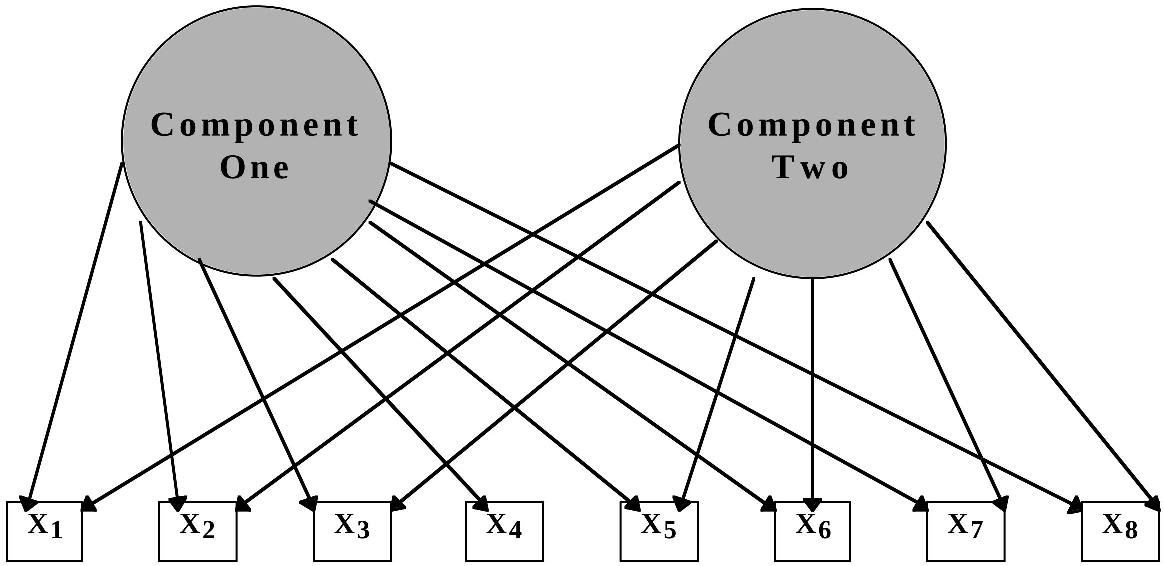
a Dependent Variable: X6

The initial two-dimensional solution using Exploratory Factor Analysis is solving the following model: $c_1 = b_{11}X_1 + b_{12}X_2 + b_{13}X_3 + b_{14}X_4 + b_{15}X_5 + b_{16}X_6 + b_{17}X_7 + b_{18}X_8$

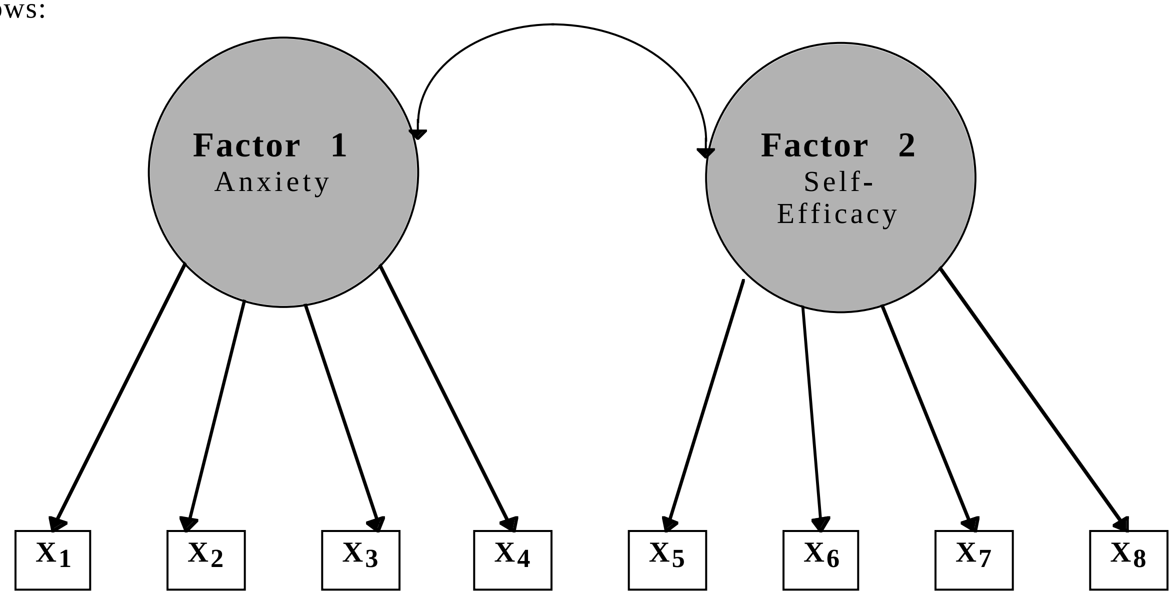
$c_2 = b_{21}X_1 + b_{22}X_2 + b_{23}X_3 + b_{24}X_4 + b_{25}X_5 + b_{26}X_6 + b_{27}X_7 + b_{28}X_8$

In matrix notation: $\mathbf{C} = \mathbf{B}\,\mathbf{X}$

Because the Components are orthogonal initially the extraction can occur sequentially.



Oblimin Rotation allows the Factors to be correlated, but attempts to keep structure orthogonal Factor correlations of zero) when the Delta parameter is set to 0. It is also an attempt to simplify the interpretation of the solution by rearranging the loadings, maximizing some weights (moving them toward 1) while also minimizing others (moving them toward 0). Although the loadings are technically not zero, if they are near zero the loading can be "ignored for the purposes of interpretation." The solution may be interpreted as follows:



Component Score Coefficient Matrix (B)

	Component	
	1	2
X1	-.068	.194
X2	-.017	.321
X3	.100	.410
X4	-.047	.301
X5	.283	.031
X6	.255	-.007
X7	.269	.012
X8	.269	.033

One reason that so many researchers choose orthogonal rotations is that they find it confusing when variables "split" across factors (i.e., have salient loadings on more than one factor). However, this is to be expected if the factors are correlated. Multiple Latent Factors may "cause" a subject to respond (behave, perform) as they do.

These are weights applied to the standardized variables in order to create Factor Scores. For example, to create Factor scores for the 3 factors (fac1_obl, fac2_obl, in the data set above), you would convert the variables to z-scores and compute:

fac1_obl= -.068(z1) -.017(z2) +.100(z3) -.047(z4) +.283(z5) +.255(z6) +.269(z7) +.269(z8)
fac2_obl= .194(z1) +.321(z2) +.410(z3) +.301(z4) +.031(z5) -.007(z6) +.012(z7) +.033(z8)

In matrix notation, the matrix of Factor score, $\mathbf{M}_{30,2} = \mathbf{Z}_{30,8}\mathbf{B}_{8,2}$

Fortunately, SPSS software computes these automatically and will save them if requested.

Scoring Coefficients as a Regression Model

The R^2 of 1.00 indicates that the Factor Scores are perfectly described by a linear combination of the variables X1 - X8.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	1.000 a	**1.000**	1.000	1.3947E-10

a Predictors: (Constant), X1, X2, X3, X4, X5, X6, X7, X8

Coefficients

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
	B	Std. Error	Beta			Zero-order	Partial	Part	Tol.	VIF
1 **(Constant)**	3.350									
X1	.207	.000	**.194**	-	-	**.621**	1.000	.150	.603	1.657
X2	.428	.000	**.321**	-	-	**.866**	1.000	.193	.362	2.762
X3	.482	.000	**.410**	-	-	**.896**	1.000	.195	.227	4.407
X4	.310	.000	**.301**	-	-	**.867**	1.000	.124	.170	5.876
X5	.046	.000	**.031**	-	-	**-.406**	1.000	.013	.166	6.041
X6	-.009	.000	**-.007**			**-.459**	-1.000	-.004	.225	4.453
X7	.014	.000	**.012**	-	-	**-.433**	1.000	.006	.273	3.665
X8	.044	.000	**.033**	-	-	**-.377**	1.000	.016	.221	4.530

a Dependent Variable: fac2_obl

Note: These Zero-Order Correlations are identical to the Structure Coefficients (**S**).

Communalities

	Initial	Extraction
X1	.397	.328
X2	.638	.651
X3	.773	.807
X4	.830	.751
X5	.834	.807
X6	.775	.696
X7	.727	.748
X8	.779	.645

Extraction Method: Principal Axis Factoring.

In a Principal Axis Factoring and Image Analysis, the initial communalities are estimated by Squared Multiple Correlations SMC; R^2). These values are used to replace the 1s in the diagonal of a correlation matrix. For example. The initial communality estimate for X8 is the R^2 from the model:

$X_8 = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 + b_7X_7 + e$

From a regression context, (1 - Tolerance) is the inital communality Estimate.

$\Sigma\lambda = \Sigma h^2_{initial} = 5.735$ $\Sigma h^2_{ext} = 5.433$ (5.433/8)x100=67.91% of the variance among the 8 variables is explained by a two-factor solution.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	4.582	57.269	57.269	4.277	53.458	53.458	3.821
2	1.411	17.639	74.908	1.157	14.459	67.917	3.077
3	.684	8.545	83.453				
4	.470	5.873	89.326				
5	.375	4.692	94.018				
6	.230	2.873	96.891				
7	.188	2.353	99.245				
8	.060	.755	100.000				

These are the actual eigenvalues when SMCs are used in the diagonal.

These values are from the PCA, SPSS reports these incorrectly

$\Sigma\lambda = 5.433$ (5.433/8) = .67917

a. When components are correlated, sum of squared loadings cannot be added to obtain a total variance.

Component Matrix[a] (U)

	Component 1	Component 2
X1	-.561	.114
X2	-.721	.364
X3	-.574	.691
X4	-.791	.353
X5	.817	.374
X6	.792	.262
X7	.803	.321
X8	.740	.312

Pattern Matrix[b] (P)

	Component 1	Component 2
X1	-.296	**.366**
X2	-.185	**.700**
X3	.213	**.983**
X4	-.245	**.720**
X5	**.917**	.039
X6	**.798**	-.069
X7	**.859**	-.012
X8	**.806**	.006

Structure Matrix (S)

	Component 1	Component 2	h^2
X1	-.475	.510	.441
X2	-.527	.791	.763
X3	-.266	.879	.848
X4	-.596	.840	.790
X5	.898	-.408	.837
X6	.832	-.459	.766
X7	.865	-.431	.800
X8	.803	-.387	.748

a. Extraction Method: Principal Axis Factoring.
b. Rotation Method: Oblimin with Kaiser Normalization.
a. 2 components extracted
b. Rotation converged in 6 iterations.

Component Correlation Matrix

Component	1	2
1	1.000	-.488
2	-.488	1.000

Note: The Correlation between the Factors scores using the two extraction methods were, r = .994 for Factor 1, and r =.989 for Factor 2.