Conversion of Common Test Statistics to \( r \) and \( d \) Values

<table>
<thead>
<tr>
<th>Statistic</th>
<th>( r )-value</th>
<th>( d )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( t )</td>
<td>( \sqrt{\frac{t^2}{(t^2 + df)}} )</td>
<td>( \frac{2t}{\sqrt{df}} )</td>
</tr>
<tr>
<td>2. ( z )</td>
<td>( \sqrt{\frac{z^2}{z^2 + N}} )</td>
<td>( \frac{2z}{\sqrt{N}} )</td>
</tr>
<tr>
<td>3. ( F ) ( df_n = 1 )</td>
<td>( \sqrt{\frac{F}{F + df_d}} )</td>
<td>( 2\sqrt{\frac{F}{df_d}} )</td>
</tr>
<tr>
<td>4. ( F ) ( df_n &gt; 1 )</td>
<td>( \sqrt{\frac{df_n F}{df_n F + df_d}} )</td>
<td>( 2\sqrt{\frac{df_n F}{df_d}} )</td>
</tr>
<tr>
<td>5. ( \chi^2 ) ( df = 1 )</td>
<td>( \sqrt{\frac{\chi^2}{N}} )</td>
<td>( 2\sqrt{\frac{\chi^2}{N - \chi^2}} )</td>
</tr>
<tr>
<td>6. ( \chi^2 ) ( df &gt; 1 )</td>
<td>( \sqrt{\frac{\chi^2}{\chi^2 + N}} )</td>
<td>( 2\sqrt{\frac{\chi^2}{N}} )</td>
</tr>
<tr>
<td>7. ( r )</td>
<td>( r )</td>
<td>( \sqrt{\frac{4r^2}{1 - r^2}} )</td>
</tr>
<tr>
<td>8. ( d )</td>
<td>( \sqrt{\frac{d^2}{4 + d^2}} )</td>
<td>( d )</td>
</tr>
</tbody>
</table>

Note. \( df_n \) = degrees of freedom for the numerator, \( df_d \) = degrees of freedom for the denominator. Tables taken after Friedman (1982) and Wolf (1986).

Cohen suggested computing:

\[
d = \frac{(\overline{Y}_1 - \overline{Y}_2)}{\sqrt{(s_1^2 + s_2^2)/2}}.\]

Hedges and Olkin (1985) suggested an adjusted \( d \),

\[
\tilde{d} = \frac{(\overline{Y}_1 - \overline{Y}_2)}{\sqrt{(s_1^2 + s_2^2)/2}} \left[ 1 - \frac{3}{4(n_1 + n_2) - 9} \right].
\]

Hedges’ \( g \) =

\[
\frac{(\overline{Y}_1 - \overline{Y}_2)}{\sqrt{((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2)/((n_1 + n_2) - 2))}} \left[ 1 - \frac{3}{4(n_1 + n_2) - 9} \right].
\]
Common Critical Values from the Normal Distribution for Quick Approximate Power Analysis

<table>
<thead>
<tr>
<th>Power</th>
<th>Distance</th>
<th>α = 0.05</th>
<th></th>
<th>α = 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1-tailed</td>
<td>2-tailed</td>
<td>1-tailed</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.525</td>
<td>2.170</td>
<td>2.485</td>
<td>2.846</td>
</tr>
<tr>
<td>0.80</td>
<td>-0.842</td>
<td>2.487</td>
<td>2.802</td>
<td>3.168</td>
</tr>
<tr>
<td>0.90</td>
<td>-1.282</td>
<td>2.927</td>
<td>3.242</td>
<td>3.608</td>
</tr>
</tbody>
</table>

(Critical Values in Parentheses)

For a 2-group design, approximate per group sample size \( (n_j) \) for a given \( \alpha \) and level of Statistical Power \( (1-\beta) \) for the can be solved as:

\[
n_j \geq \frac{2z_{cv}^2}{d^2}, \text{ where } z_{cv}^2 \text{ is the critical value from the Table above, } d \text{ is a standardized mean difference, } d = \frac{(\bar{Y}_1 - \bar{Y}_2)}{s}, \text{ and } s \text{ is an assumed standard deviation. As pointed out above, various metrics have been proposed. In general, the use of Cohen’s } d \text{ the adjusted } d, \text{ or Hedges’ } g \text{ will lead to approximately the same result.}
\]

For example, suppose a study reports the control group had a mean of \( \bar{Y}_c = 11.8 \), the treatment group had a mean of \( \bar{Y}_t = 12.7 \) and the pooled standard deviation was \( s = 1.5 \). Then the standardized mean difference would be: \( d = (12.7-11.8)/1.5 = 0.6 \).

For a future study to have **70% Power** \( (1-\beta = 0.70) \) for a 2-tailed test at \( \alpha = 0.05 \) The approximate necessary per group sample size would be:

\[
n_j \geq \frac{2(2.485^2)}{0.6^2} \geq \frac{2(6.175225)}{0.36} \geq 34.3 \approx 35.
\]

```
proc power;
 onewayanova test=overall
   alpha=0.05
   groupmeans = 11.8 | 12.7
   stddev = 1.5
   npergroup = .
   power = .7; run;
```

```
Overall F Test for One-Way ANOVA
Fixed Scenario Elements

    Method       Exact
    Alpha        0.05
    Group Means  11.8 12.7
    Standard Deviation  1.5
    Nominal Power     0.7

Computed N Per Group

    Actual  N Per Power  Group
          0.709    36
```
For a future study to have \textbf{80\% Power (1-\( \beta \) = 0.80)} for a 2-tailed test at \( \alpha = 0.01 \)

The approximate necessary per group sample size would be:

\[ n_j \geq \frac{2(3.418^2)}{0.6^2} \geq \frac{2(11.682724)}{0.36} \geq 64.9 \approx 65. \]

\begin{verbatim}
proc power;
onewayanova test=overall
  alpha=0.01
  groupmeans = 11.8 | 12.7
  stddev = 1.5
  npergroup = .
power = .8; run;
\end{verbatim}

Reversing this process, if a researcher knew that he could only obtain 100 total subjects (\( n_j = 50 \) per group), then we could solve for an approximate minimum effect size (\( d \)):

\[ d \geq \frac{z_{cv}}{\sqrt{n_j/2}}. \]

Thus, if the research desired \textbf{80\% Power (1-\( \beta \) = 0.80)} for a 2-tailed test at \( \alpha = 0.05 \)

\[ d \geq \frac{2.802}{\sqrt{50}} \geq \frac{2.802}{5} \geq 0.5604 \] would be the approximate necessary effect size.

To double check this enter the effect size of \( d = 0.5604 \) the critical value for \textbf{80\% Power (1-\( \beta \) = 0.80)} for a 2-tailed test at \( \alpha = 0.05 \) into

\[ n_j \geq \frac{2z_{cv}^2}{d^2} \geq \frac{2(2.802^2)}{0.5604^2} \geq \frac{15.702408}{0.31404816} \geq 50 \]

\begin{verbatim}
proc power;
onewayanova test=overall
  alpha=0.05
  groupmeans = 0 | 0.5604
  stddev = 1
  npergroup = 50
  power = .; run;
\end{verbatim}

\begin{verbatim}
    The POWER Procedure
    Overall F Test for One-Way ANOVA
    Fixed Scenario Elements
    Method            Exact
    Alpha             0.01
    Group Means       11.8 12.7
    Standard Deviation 1.5
    Nominal Power     0.8

    Computed N Per Group
    Actual N Per Power Group
    0.803       67
\end{verbatim}

\begin{verbatim}
    The POWER Procedure
    Overall F Test for One-Way ANOVA
    Fixed Scenario Elements
    Method            Exact
    Alpha             0.05
    Group Means       0 0.5604
    Standard Deviation 1
    Sample Size Per Group 50

    Computed Power
    Power
    0.792
\end{verbatim}