data ptsd;
input id ptsd agetrma agetrma2 viotrma;
cards;
 1 1 12 12 1
 2 1 10 10 1
 3 1 16 16 0
 4 1 14 14 1
 5 1 14 17 1
 6 0 30 30 1
 7 0 21 21 1
 8 0 17 14 0
 9 0 25 25 1
10 0 30 30 0
11 0 29 29 0
12 0 19 19 0
13 0 24 24 1
14 0 27 27 0
15 0 19 19 0
16 0 26 26 0
17 0 31 31 1
18 0 22 22 0
19 0 28 28 0
20 0 35 35 1;

proc freq data=ptsd;
tables ptsd*viotrma/chisq relrisk;run;

proc logistic data=ptsd descending;
model ptsd = /rsquare clodds=wald;run;

proc logistic data=ptsd descending;
model ptsd = viotrma /rsquare clodds=wald;run;
<table>
<thead>
<tr>
<th></th>
<th>ptsd</th>
<th>viotrma</th>
<th>Frequency</th>
<th>Percent</th>
<th>Row Pct</th>
<th>Col Pct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>45.00</td>
<td>30.00</td>
<td>75.00</td>
<td>75.00</td>
<td>75.00</td>
<td>75.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60.00</td>
<td>40.00</td>
<td></td>
<td>60.00</td>
<td>60.00</td>
<td>60.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90.00</td>
<td>60.00</td>
<td></td>
<td>90.00</td>
<td>90.00</td>
<td>90.00</td>
<td></td>
</tr>
<tr>
<td>1/10 that experienced Violent Trauma developed PTSD</td>
<td>Predicted Probability = 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.00</td>
<td>20.00</td>
<td>25.00</td>
<td>25.00</td>
<td>25.00</td>
<td>25.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.00</td>
<td>80.00</td>
<td></td>
<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.00</td>
<td>40.00</td>
<td></td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
<td></td>
</tr>
<tr>
<td>4/10 that experienced Violent Trauma developed PTSD</td>
<td>Predicted Probability = 0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With No predictor 5 out 20 cases had PTSD  Predicted Probability = .25
Odds Ratio = (.25/(1-.25)) = .3333
Log Odds = ln(.3333) = -1.0986

With No predictors (Base Rate) Odds are that patients are 3 times as likely to NOT have PTSD

With Viotrma as a predictor The Odds Ratio = (0.4/(1-0.4)) = 6
(0.1/(1-0.1))

If patient experienced a Violent Trauma they are 6 times as likely to develop PTSD
Statistics for Table of ptsd by viotrma

<table>
<thead>
<tr>
<th>Statistic</th>
<th>DF</th>
<th>Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>1</td>
<td>2.4000</td>
<td>0.1213</td>
</tr>
<tr>
<td>Likelihood Ratio Chi-Square</td>
<td>1</td>
<td>2.5315</td>
<td>0.1116</td>
</tr>
<tr>
<td>Phi Coefficient</td>
<td></td>
<td>0.3464</td>
<td></td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td></td>
<td>0.3464</td>
<td>0.1949</td>
</tr>
</tbody>
</table>

The FREQ Procedure

Statistics for Table of ptsd by viotrma

Estimates of the Relative Risk (Row1/Row2)

<table>
<thead>
<tr>
<th>Type of Study</th>
<th>Value</th>
<th>95% Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-Control (Odds Ratio)</td>
<td>6.0000</td>
<td>0.5322</td>
</tr>
<tr>
<td>Cohort (Col1 Risk)</td>
<td>3.0000</td>
<td>0.4954</td>
</tr>
<tr>
<td>Cohort (Col2 Risk)</td>
<td>0.5000</td>
<td>0.2340</td>
</tr>
</tbody>
</table>

Sample Size = 20
The LOGISTIC Procedure

Model Information
Data Set                      WORK.PTSD
Response Variable             ptsd
Number of Response Levels    2
Model                         binary logit
Optimization Technique       Fisher's scoring

Number of Observations Read          20
Number of Observations Used          20

Response Profile
Ordered           Total
Value  ptsd   Frequency
1    1   5
2    0   15

Probability modeled is ptsd=1.

-2 Log L = 22.493

Analysis of Maximum Likelihood Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DF</th>
<th>Estimate</th>
<th>Error</th>
<th>Chi-Square</th>
<th>Pr &gt; ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>-1.0986</td>
<td>0.5164</td>
<td>4.5261</td>
<td>0.0334</td>
</tr>
</tbody>
</table>

Predicted Log Odds are -1.0986

Predicted Probabilities
\[ \hat{P} = \frac{1}{1+e^{-(b_0)}} = 1/1+e^{1.0986} = 0.25 \]
Model Information

Data Set                      WORK.PTSD
Response Variable             ptsd
Number of Response Levels     2
Model                        binary logit
Optimization Technique       Fisher’s scoring

Response Profile

<table>
<thead>
<tr>
<th>Ordered Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>ptsd</td>
<td>Frequency</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
</tbody>
</table>

Probability modeled is ptsd=1.

Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

Model Fit Statistics

<table>
<thead>
<tr>
<th>Intercept and Criterion</th>
<th>Intercept Only</th>
<th>Covariates</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>24.493</td>
<td>23.962</td>
</tr>
<tr>
<td>SC</td>
<td>25.489</td>
<td>25.953</td>
</tr>
<tr>
<td>-2 Log L</td>
<td>22.493</td>
<td>19.962</td>
</tr>
</tbody>
</table>

R-Square 0.1189 Max-rescaled R-Square 0.1761

Testing Global Null Hypothesis: BETA=0

<table>
<thead>
<tr>
<th>Test</th>
<th>Chi-Square</th>
<th>DF</th>
<th>Pr &gt; ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood Ratio</td>
<td>2.5315</td>
<td>1</td>
<td>0.1116</td>
</tr>
<tr>
<td>Score</td>
<td>2.4000</td>
<td>1</td>
<td>0.1213</td>
</tr>
<tr>
<td>Wald</td>
<td>2.1014</td>
<td>1</td>
<td>0.1472</td>
</tr>
</tbody>
</table>
The LOGISTIC Procedure

Analysis of Maximum Likelihood Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DF</th>
<th>Estimate</th>
<th>Error</th>
<th>Chi-Square</th>
<th>Pr &gt; ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
<td>-2.1972</td>
<td>1.0541</td>
<td>4.3450</td>
<td>0.0371</td>
</tr>
<tr>
<td>viotrma</td>
<td>1</td>
<td>1.7918</td>
<td>1.2360</td>
<td>2.1014</td>
<td>0.1472</td>
</tr>
</tbody>
</table>

Odds Ratio Estimates

\[
\ln \left( \frac{P(Y=1)}{P(Y=0)} \right) = -2.1972 + 1.7918X
\]

<table>
<thead>
<tr>
<th>Effect</th>
<th>Estimate</th>
<th>95% Wald Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>viotrma</td>
<td>6.000</td>
<td>0.532  67.649</td>
</tr>
</tbody>
</table>

Association of Predicted Probabilities and Observed Responses

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Concordant</td>
<td>48.0</td>
<td>Somers' D</td>
</tr>
<tr>
<td>Percent Discordant</td>
<td>8.0</td>
<td>Gamma</td>
</tr>
<tr>
<td>Percent Tied</td>
<td>44.0</td>
<td>Tau-a</td>
</tr>
<tr>
<td>Pairs</td>
<td>75</td>
<td>c</td>
</tr>
</tbody>
</table>

Wald Confidence Interval for Odds Ratios

\[
e^{1.7918} = 6.0 = \text{Odds Ratio}
\]

\[
\text{Lower Bound} = e^{(1.7918 - [1.96*1.236])} = 0.532
\]

\[
\text{Upper Bound} = e^{(1.7918 + [1.96*1.236])} = 67.649
\]

Predicted Probabilities

\[
\hat{P} = \frac{1}{1+e^{-(b_0+b_1X)}}
\]

For Viotrma = 0 \( \hat{P} = \frac{1}{1+e^{-(-2.1972+1.7918(1))}} = 0.10 \) \((.1/9) = 0.11\)

For Viotrma = 1 \( \hat{P} = \frac{1}{1+e^{-(-2.1972+1.7918(1))}} = 0.40 \) \((.4/6) = 0.67\)

Odds Ratio = \(0.67/0.11 = 6.0\)
Why -2 Log Likelihood?

\[ \mathcal{L} = \prod_{i=1}^{N} \text{Prob}(Y_i) \]

By definition: \( \text{Prob}(Y_i = 1) = \hat{P}_i \) and \( \text{Prob}(Y_i = 0) = 1 - \hat{P}_i \)

\[ \mathcal{L}(Y_i|\hat{P}) = \prod_{i=1}^{N} [\hat{P}_i^{Y_i} (1 - \hat{P}_i)^{(1-Y_i)}] \]

\[-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -2 \sum_{i=1}^{N} Y_i [\ln(\hat{P}_i)] + (1 - Y_i) [\ln(1 - \hat{P}_i)] \]

For Null Model with no Predictors all cases have the same predicted probability, \( \hat{P}_i = 0.25 \) for all \( i \).

\[
\begin{align*}
1 \times \ln(0.25) &= -1.3863 & 0 \\
1 \times \ln(0.25) &= -1.3863 & 0 \\
1 \times \ln(0.25) &= -1.3863 & 0 \\
1 \times \ln(0.25) &= -1.3863 & 0 \\
1 \times \ln(0.25) &= -1.3863 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
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1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
1 \times \ln(0.75) &= -0.2876 & 0 \\
-2 \times (-6.93147) + & 4.31523 = 22.49340578
\end{align*}
\]

This is the -2 Log Likelihood of the Intercept Only (Null) Model.
Using the Predicted Values of 0.40 for Viotrma = 1 and 0.10 for Viotrma = 0

| Viotram | $\hat{P}_i$ | $-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -2 \sum_{i=1}^{N} Y_i[\ln(\hat{P}_i)] + (1 - Y_i)[\ln(1 - \hat{P}_i)]$ |
|---------|-------------|-------------------------------------------------------------------------------------------------|
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 0       | 0.1         | 1 * ln(0.1) = -2.3026                                                                            |
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 1       | 0.4         | 1 * ln(0.4) = -0.9163                                                                            |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 0       | 0.1         | 1 * ln(0.9) = -0.1054                                                                          |
| 1       | 0.4         | 1 * ln(0.6) = -0.5108                                                                          |

$-2 \times (-5.9678 + + -4.0132) = 19.962$

This is the -2 Log Likelihood of the One Predictor Model

Likelihood ratio Chi Square is the Difference in -2 Log Likelihood for each model

$\chi^2 = 22.493 - 19.962 = 2.531$
Why -2 Log Likelihood?

\[ \mathcal{L}(Y_i|\hat{P}_i) = \prod_{i=1}^{N} [\hat{P}_i^{Y_i} (1 - \hat{P}_i)^{(1-Y_i)}] \]

\[-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -2 \sum_{i=1}^{N} Y_i [\ln(\hat{P}_i)] + (1 - Y_i) [\ln(1 - \hat{P}_i)] \]

When \( Y = 0 \) then this zeroes out

So it’s only used when \( Y = 1 \); thus a summation over \( n_1 \),= Number with \( Y = 1 \)

\[-2 \left[ \sum_{i=1}^{n_1} Y_i \ln(\hat{P}_i) + \sum_{i=1}^{n_0} (1 - Y_1) [\ln(1 - \hat{P}_i)] \right] \]

When \( Y = 1 \) then this zeroes out

So it’s only used when \( Y = 0 \); thus a summation over \( n_0 \),= # with \( Y = 0 \)

\[-2 \left[ \sum_{i=1}^{n_1} 1 \ln(\hat{P}_i) + \sum_{i=1}^{n_0} 1 [\ln(1 - \hat{P}_i)] \right] \]

\[-2 \left[ \sum_{i=1}^{n_1} \ln(\hat{P}_i) + \sum_{i=1}^{n_0} [\ln(1 - \hat{P}_i)] \right] \]

You can also get to this step by noting there will be \( n_1 \) values with a value of \( Y = 1 \) There will be \( n_0 \) values with a value of \( Y = 0 \); thus

\[ \mathcal{L}(Y_i|\hat{P}_i) = \prod_{i=1}^{n_1} \hat{P}_i \prod_{i=1}^{n_0} (1 - \hat{P}_i) \] . Thus

\[-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -2 \left[ \sum_{i=1}^{n_1} \ln(\hat{P}_i) + \sum_{i=1}^{n_0} [\ln(1 - \hat{P}_i)] \right] \]

\[-\left[ \sum_{i=1}^{n_1} 2 \ln(\hat{P}_i) + \sum_{i=1}^{n_0} 2 [\ln(1 - \hat{P}_i)] \right] \]

\[-\left[ \sum_{i=1}^{n_1} \ln((0 - \hat{P}_i)^2) + \sum_{i=1}^{n_0} [\ln(1 - \hat{P}_i)^2] \right] \]

Similar to a Sums of Squares calculation but reversed.
**Why -2 Log Likelihood?**

\[ \mathcal{L}(Y_i|\hat{P}_i) = \prod_{i=1}^{N}[\hat{P}_i^{Y_i}(1 - \hat{P}_i)^{(1 - Y_i)}] \]

\[-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -2 \sum_{i=1}^{N} Y_i[\ln(\hat{P}_i)] + (1 - Y_i)[\ln(1 - \hat{P}_i)] \]

\[-[\sum_{i=1}^{n_1} \ln[(0 - \hat{P}_i)^2] + \sum_{i=1}^{n_0} \ln(1 - \hat{P}_i)^2] \]

For Null Model with no Predictors all cases have the same predicted probability, \( \hat{P}_i = 0.25 \) for all \( i \).

\[
2*\ln(0.25) = \rightarrow 1*\ln(0.25^2) = -2.7726 \\
1 * \ln(0.25^2) = -2.7726 \\
1 * \ln(0.25^2) = -2.7726 \\
1 * \ln(0.25^2) = -2.7726 \\
0 \\
0 \\
1 * \ln(0.75^2) = -0.57536 \\
1 * \ln(0.75^2) = -0.57536 \\
1 * \ln(0.75^2) = -0.57536 \\
1 * \ln(0.75^2) = -0.57536 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
-(-13.862 + -8.63046) = 22.49340578
\]

This is the -2 Log Likelihood of the Intercept Only (Null) Model.
For Null Model with no Predictors all cases have the same predicted probability, 
\( \hat{P}_i = 0.25 \) for all \( i \).

And therefore the likelihood equation
\[
\mathcal{L}(Y_i|\hat{P}_i) = \Pi_{i=1}^{N} [\hat{P}_i^{Y_i} (1 - \hat{P}_i)^{(1-y_i)}]
\]
can be written as:
\[
\mathcal{L}(Y_i|\hat{P}_i) = [\hat{P}_i^{n_1} (1 - \hat{P}_i)^{n_0}]
\]
\[
= 0.25^{5} * 0.75^{15} = 0.000976563 * 0.013363461
\]
\[
= 0.00001305025489
\]

\[-2 \ln [\mathcal{L}(Y_i|\hat{P}_i)] = -2 \ln(0.00001305025489) = 22.49340578\]

Also,
\[
-2 \ln \mathcal{L}(Y_i|\hat{P}_i) = -[\sum_{i=1}^{n_1} \ln[(0 - \hat{P}_i)^2] + \sum_{i=1}^{n_0} \ln(1 - \hat{P}_i)^2]
\]

Reduces to:
\[
= -[n_1 \ln(\hat{P}_i^2) + n_0 \ln(1 - \hat{P}_i)^2]
\]
\[
= -[5 \ln(0.25^{2}) + 15(0.75^{2})]
\]
\[
= -[5(-2.7726)+ 15(-.57536)]
\]
\[
= -[13.8629436 + 8.63046217]
\]
\[
= 22.49340578
\]