Computational Example of Pearson Correlation and Simple Regression Solution.

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\begin{array}{ccccccc}
Y - \overline{Y} & (Y - \overline{Y})^2 & Z_Y = (Y-\overline{Y})/S_Y & Z_YZ_X & Z_X = (X-\overline{X})/S_X & (X - \overline{X}) & (X - \overline{X})^2 \\
56 - 50 & 6 & 36 & 6/4 = 1.50 & 0 & 0 = 0/2 & (32 - 32) & 0 \\
55 - 50 & 5 & 25 & 5/4 = 1.25 & 1.875 & 1.5 = 3/2 & (35 - 32) & 3 \\
52 - 50 & 2 & 4 & 2/4 = 0.50 & -0.250 & -0.5 = -1/2 & (31 - 32) & -1 \\
49 - 50 & -1 & 1 & -1/4 = -0.25 & -0.375 & 1.5 = 3/2 & (35 - 32) & 3 \\
49 - 50 & -1 & 1 & -1/4 = -0.25 & 0.250 & -1.0 = -2/2 & (30 - 32) & -2 \\
45 - 50 & -5 & 25 & -5/4 = -1.25 & 0 & 0 = 0/2 & (32 - 32) & 0 \\
46 - 50 & -4 & 16 & -4/4 = -1.00 & 1.000 & -1.0 = -2/2 & (30 - 32) & -2 \\
48 - 50 & -2 & 4 & -2/4 = -0.50 & 0.250 & -0.5 = 1/2 & (31 - 32) & -1 \\
\end{array}
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\(\overline{Y} = 50\sum(Y - \overline{Y})^2 = 112\) \(\overline{X} = 32\) \(\sum(X - \overline{X})^2 = 28\)

Mean of Y \(\Sigma Z_YZ_X = 2.75\) Mean of X

\(S_Y^2 = \Sigma(Y - \overline{Y})^2/(N-1) = 112/7 = 16\) \(S_X^2 = \Sigma(X - \overline{X})^2/(N-1) = 28/7 = 4\)

Variance of Y \(\Sigma Z_YZ_X = 2.75\) \(N-1) = 0.393\)

Pearson Product-Moment Correlation Coefficient \(r = \Sigma Z_YZ_X/(N - 1) = 2.75/7 = 0.393\)

Least Squares Regression Solution takes this Form:

\(\hat{Y} = a + bX\)

where, \(b = r(S_Y/S_X) = 0.393(4/2) = 0.786\)

and

\(a = \overline{Y} - b\overline{X} = 50 - (.786)32 = 24.857\).

Thus the regression solution is the following linear transformation of X:

\(\hat{Y} = 24.857 + 0.786X\)

To test the statistical significance of a simple regression solution one can use either \(t_{N-2} = \sqrt{r^2/(1 - r^2)}\) \(t_6 = \sqrt{0.154/(1 - 0.154)} = 1.046\) This value can be check against a critical value from a t-distribution table with \((N-2)\) degrees-of-freedom. For example, from Agresti & Finlay (1997) Table B, p. 669, the t-distribution with 6 degrees-of-freedom \((df)\) has a critical value of 2.447 for a two-tailed test for \(\alpha = .05\) (i.e., \(t_{0.025}\)) level of statistical significance. Since the calculated value does not exceed this critical value, the null hypothesis \((H_0: \rho = 0)\) is NOT rejected. There is not sufficient evidence to claim a reliable relationship between Y and X. Concomitantly, X and Y do not share a statistically significant amount of variance \((H_0: \rho^2 = 0)\) or X is not a statistically significant predictor \((H_0: \beta = 0)\) of Y.

Pearson Correlation: \( r = 0.393, \ t(6) = 1.046, \) 2-tailed \( p \)-value = 0.336.

Raw-Score Regression Solution: \( \hat{Y} = 24.857 + 0.786X \)

Standardized Regression Solution: \( \hat{Z}_Y = 24.857 + 0.786Z_X \)