Profile Analysis Example

In the One-Way Repeated Measures ANOVA, two factors represent separate sources of variance. Their interaction presents an independent sources of error variation. Suppose a design in which a Factor A has four levels.

<table>
<thead>
<tr>
<th>Person</th>
<th>Information (A1)</th>
<th>Vocabulary (A2)</th>
<th>Digit Span (A3)</th>
<th>Digit Symbol (A4)</th>
<th>( \bar{Y}_{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>127</td>
<td>103</td>
<td>97</td>
<td>113</td>
<td>110</td>
</tr>
<tr>
<td>P2</td>
<td>123</td>
<td>127</td>
<td>121</td>
<td>89</td>
<td>115</td>
</tr>
<tr>
<td>P3</td>
<td>111</td>
<td>127</td>
<td>97</td>
<td>113</td>
<td>112</td>
</tr>
<tr>
<td>P4</td>
<td>99</td>
<td>95</td>
<td>121</td>
<td>89</td>
<td>101</td>
</tr>
<tr>
<td>P5</td>
<td>99</td>
<td>95</td>
<td>89</td>
<td>81</td>
<td>91</td>
</tr>
<tr>
<td>P6</td>
<td>95</td>
<td>103</td>
<td>89</td>
<td>73</td>
<td>90</td>
</tr>
<tr>
<td>P7</td>
<td>87</td>
<td>87</td>
<td>81</td>
<td>81</td>
<td>84</td>
</tr>
<tr>
<td>P8</td>
<td>83</td>
<td>87</td>
<td>81</td>
<td>73</td>
<td>81</td>
</tr>
</tbody>
</table>

\( \bar{Y}_{1} = \bar{Y}_{*1} = 103 \), \( \bar{Y}_{*2} = 103 \), \( \bar{Y}_{*3} = 97 \), \( \bar{Y}_{*4} = 89 \), \( \bar{Y}_{**} = 98 \)

ANOVA MODEL: \( Y_{ijk} = \mu^{**} + \alpha_{j} + \pi_{i} + \pi \alpha_{ij} + \varepsilon_{ij} \)

Factor A has four marginal means, \( \bar{Y}_{1}, \bar{Y}_{2}, \bar{Y}_{3}, \) and \( \bar{Y}_{4} \), and (A=1=4-1=3) degrees of freedom. The null hypothesis for Factor A is \( H_{0}: \mu_{*1} = \mu_{*2} = \mu_{*3} = \mu_{*4} \) or \( H_{0}: \Sigma \alpha_{j}^{2} = 0 \).

Factor P is a Between-Subjects factor that estimates the individual differences among the subjects. The Sum of Squares of the subject effect (\( \Sigma \pi_{i}^{2} \)) is a source of variance that is separate from the Within-Subjects Factor A.

Because Factor A is a within-subjects factor, the Sum of Squares for the Repeated Measures by Subjects interaction (\( \Sigma \pi \alpha_{ij}^{2} \)) is a source of error.

The computation of the Sums of Squares (SS) for the main effects of Factors A and P are similar to the two-way analysis. Marginal means are subtracted from the grand mean, squared, weighted by the marginal sample size, and summed.

SPSS Output from GLM

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information</td>
<td>103.00</td>
<td>16.00</td>
<td>8</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>103.00</td>
<td>16.00</td>
<td>8</td>
</tr>
<tr>
<td>Digit Span</td>
<td>97.00</td>
<td>16.00</td>
<td>8</td>
</tr>
<tr>
<td>Digit Symbol</td>
<td>89.00</td>
<td>16.00</td>
<td>8</td>
</tr>
</tbody>
</table>

Multivariate Tests

<table>
<thead>
<tr>
<th>Effect</th>
<th>Value</th>
<th>F</th>
<th>Hypothesis df</th>
<th>Error df</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA</td>
<td>Pillai’s Trace</td>
<td>.698</td>
<td>3.859a</td>
<td>3.000</td>
<td>5.000</td>
</tr>
<tr>
<td></td>
<td>Wilks’ Lambda</td>
<td>.302</td>
<td>3.859a</td>
<td>3.000</td>
<td>5.000</td>
</tr>
<tr>
<td></td>
<td>Hotelling’s Trace</td>
<td>2.315</td>
<td>3.859a</td>
<td>3.000</td>
<td>5.000</td>
</tr>
<tr>
<td></td>
<td>Roy’s Largest Root</td>
<td>2.315</td>
<td>3.859a</td>
<td>3.000</td>
<td>5.000</td>
</tr>
</tbody>
</table>

a. Exact statistic
Measure: WISC

Mauchly’s Test of Sphericity

<table>
<thead>
<tr>
<th>Within Subject Effect</th>
<th>Mauchly’s W</th>
<th>Approx. Chi-Square</th>
<th>df</th>
<th>Sig.</th>
<th>Epsilon&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Greenhouse-Geisser</th>
<th>Huynh-Feldt</th>
<th>Lower-Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA</td>
<td>.625</td>
<td>2.689</td>
<td>5</td>
<td>.751</td>
<td>.782</td>
<td>1.00</td>
<td>.333</td>
<td></td>
</tr>
</tbody>
</table>

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom of the averaged tests of significance.

b. Design: Intercept+FACTORA Within Subject Design: FACTORB

Residual SSCP Matrix

<table>
<thead>
<tr>
<th>Information</th>
<th>Vocabulary</th>
<th>Digit Span</th>
<th>Digit Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum-of-Squares and Cross-Products</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td>1792.000</td>
<td>1312.000</td>
<td>1056.000</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>1312.000</td>
<td>1792.000</td>
<td>960.000</td>
</tr>
<tr>
<td>Digit Span</td>
<td>1056.000</td>
<td>960.000</td>
<td>1792.000</td>
</tr>
<tr>
<td>Digit Symbol</td>
<td>1376.000</td>
<td>1024.000</td>
<td>576.000</td>
</tr>
<tr>
<td>Covariance</td>
<td>256.000</td>
<td>187.429</td>
<td>150.857</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>187.429</td>
<td>256.000</td>
<td>137.143</td>
</tr>
<tr>
<td>Digit Span</td>
<td>150.857</td>
<td>137.143</td>
<td>256.000</td>
</tr>
<tr>
<td>Digit Symbol</td>
<td>196.571</td>
<td>146.286</td>
<td>82.286</td>
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<tr>
<td>Correlations</td>
<td>1.000</td>
<td>.732</td>
<td>.589</td>
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<tr>
<td>Vocabulary</td>
<td>.732</td>
<td>1.000</td>
<td>.536</td>
</tr>
<tr>
<td>Digit Span</td>
<td>.589</td>
<td>.536</td>
<td>1.000</td>
</tr>
<tr>
<td>Digit Symbol</td>
<td>.768</td>
<td>.571</td>
<td>.321</td>
</tr>
</tbody>
</table>

Calculation of Epsilon

\[
\hat{\varepsilon} = \frac{A^2 (\bar{s}_{ij} - \bar{s}_{*})^2}{(A - 1) \left( \sum \sum s_{ij}^2 - 2A \sum s_{i}^2 + A^2 \bar{s}_{*}^2 \right)}
\]

where \( s_{ij} \) is any element of the covariance matrix,
\( \bar{s}_{i} = \frac{(256+256+256+256)}{4} = 256 \) is the mean of variances,
\( \bar{s}_{*} = \frac{(2825.144)}{16} = 176.5714 \) is the mean of all elements, and
\( \bar{s}_{i} \) is the mean of the \( i^{th} \) row of the covariance matrix. Thus,
\( \bar{s}_{1} = \frac{(256.000+187.429+150.857+196.571)}{4} = 197.7140, \)
\( \bar{s}_{2} = \frac{(187.429+256.000+137.143+146.286)}{4} = 181.7145, \)
\( \bar{s}_{3} = \frac{(150.857+137.143+256.000+82.286)}{4} = 156.5715 \), and
\( \bar{s}_{4} = \frac{(196.571+146.286+82.286+256.000)}{4} = 170.2858. \)

\( \hat{\varepsilon} = \frac{[(16)(256-176.5714)^2]}{[(3)[(549156.81)-((8)(125622.86))+(16)(31177.473)]]} = 0.782 \)
SPSS Output from GLM (continued)

Measure: WISC  
**Tests of Within-Subjects Effects**

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphericity Assumed</td>
<td>1056.000</td>
<td>3</td>
<td>352.000</td>
<td>3.324</td>
<td>.039</td>
</tr>
<tr>
<td>Greehouse-Geisser</td>
<td>1056.000</td>
<td>2.347</td>
<td>449.979</td>
<td>3.324</td>
<td>.055</td>
</tr>
<tr>
<td>Huynh-Feldt</td>
<td>1056.000</td>
<td>3.000</td>
<td>352.000</td>
<td>3.324</td>
<td>.039</td>
</tr>
<tr>
<td>Lower-Bound</td>
<td>1056.000</td>
<td>1.000</td>
<td>1056.000</td>
<td>3.324</td>
<td>.111</td>
</tr>
<tr>
<td>ERROR(FACTORA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphericity Assumed</td>
<td>2224.000</td>
<td>21</td>
<td>105.905</td>
<td>3.324</td>
<td>.055</td>
</tr>
<tr>
<td>Greehouse-Geisser</td>
<td>2224.000</td>
<td>16.427</td>
<td>135.383</td>
<td>3.324</td>
<td>.055</td>
</tr>
<tr>
<td>Huynh-Feldt</td>
<td>2224.000</td>
<td>21.000</td>
<td>105.905</td>
<td>3.324</td>
<td>.055</td>
</tr>
<tr>
<td>Lower-Bound</td>
<td>2224.000</td>
<td>7.000</td>
<td>317.714</td>
<td>3.324</td>
<td>.055</td>
</tr>
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</table>

Measure: WISC  
**Tests of Within-Subjects Contrasts**

<table>
<thead>
<tr>
<th>Effect</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1 vs. Level 2</td>
<td>.000</td>
<td>1</td>
<td>.000</td>
<td>.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Level 2 vs. Level 3</td>
<td>288.000</td>
<td>1</td>
<td>288.000</td>
<td>1.212</td>
<td>.307</td>
</tr>
<tr>
<td>Level 3 vs. Level 4</td>
<td>512.000</td>
<td>1</td>
<td>512.000</td>
<td>1.474</td>
<td>.264</td>
</tr>
<tr>
<td>ERROR(FACTORA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1 vs. Level 2</td>
<td>960.000</td>
<td>7</td>
<td>137.143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 2 vs. Level 3</td>
<td>1664.000</td>
<td>7</td>
<td>237.714</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 3 vs. Level 4</td>
<td>2432.000</td>
<td>7</td>
<td>347.429</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This was completed by using the Contrast Option (Repeated)

Measure: WISC  
**Tests of Between-Subjects Effects**

<table>
<thead>
<tr>
<th>Effect</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>76832.000</td>
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<td>76832.000</td>
<td>435.133</td>
<td>.000</td>
</tr>
<tr>
<td>ERROR</td>
<td>1236.000</td>
<td>7</td>
<td>176.571</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Measure: WISC  
**Pairwise Comparisons**

<table>
<thead>
<tr>
<th>(I) FACTORA</th>
<th>(J) FACTORA</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig.</th>
<th>95% Confidence Interval for Difference(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>.000</td>
<td>4.140</td>
<td>1.000</td>
<td>-14.987</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-6.000</td>
<td>5.127</td>
<td>.861</td>
<td>-25.732</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>.000</td>
<td>3.854</td>
<td>.049</td>
<td>-32.958</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6.000</td>
<td>5.451</td>
<td>.890</td>
<td>-31.855</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>-6.000</td>
<td>5.237</td>
<td>.177</td>
<td>-25.732</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>14.000</td>
<td>6.590</td>
<td>.841</td>
<td>-32.958</td>
</tr>
</tbody>
</table>

Based on estimated marginal means
\(^a\). The mean difference is significant at the .05 level.

\(^a\). Adjustment for multiple comparisons: Sidak.
### Computations

Within-Subjects ANOVA Source Table: \( Y_{ijk} = \mu^{**} + \alpha_j + \pi_i + \pi\alpha_{ij} + \epsilon_{ij} \)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BETWEEN-SUBJECTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor ( P ) (( \Sigma \pi_i^2 ))</td>
<td>( \Sigma A(\overline{Y}_{p*} - \overline{Y}^{**})^2 )</td>
<td>(N - 1)</td>
<td>SS( P )/(N-1)</td>
<td></td>
</tr>
<tr>
<td>( p_1 = 12 )</td>
<td>4(110 - 98)^2</td>
<td>(8-1) = 7</td>
<td>1236/7 = 176.57</td>
<td></td>
</tr>
<tr>
<td>( p_2 = 17 )</td>
<td>+ 4(115 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_3 = 14 )</td>
<td>+ 4(112 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_4 = 3 )</td>
<td>+ 4(101 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_5 = -7 )</td>
<td>+ 4( 91 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_6 = -8 )</td>
<td>+ 4( 90 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_7 = -14 )</td>
<td>+ 4( 84 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_8 = -17 )</td>
<td>+ 4( 81 - 98)^2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **WITHIN-SUBJECTS**           |                                                      |    |             |       |
| FACTOR \( A \) (\( \Sigma \alpha_k^2 \)) | \( \Sigma n(\overline{Y}_{*A} - \overline{Y}^{**})^2 \) | (A - 1) | SS\( A \)/(A-1) | MS\( A \)/MS\( A \times P \) |
| \( a_1 = 5 \)                 | 8(103 - 98)^2                                        | (4 - 1) = 3 | 1056/3 = 352 |       |
| \( a_2 = 5 \)                 | + 8(103 - 98)^2                                      |       |             |       |
| \( a_3 = -1 \)                | + 8( 97 - 98)^2                                      |       |             |       |
| \( a_4 = -9 \)                | + 8( 89 - 98)^2                                      |       |             |       |

| Within-Subjects (Error) (\( \Sigma \pi\alpha_{ij} \)) | \( \Sigma (Y_{ij} - \overline{Y}_{p*} - \overline{Y}_{*A} + \overline{Y}^{**})^2 \) | (A-1)(N-1) | SS\( A \times P \)/df\( A \times P \) |       |
| \( pa_{11} = 12 \)            | (127-110-103+98)^2                                    | (4-1)(8-1) = 21 | (2224/21 = 105.905) |       |
| \( pa_{21} = 3 \)             | + (123-115-103+98)^2                                   |       |             |       |
| \( \ldots \)                  | + (103-110-103+98)^2                                   |       |             |       |
| \( pa_{12} = -12 \)           | + (127-115-103+98)^2                                   |       |             |       |
| \( pa_{22} = 7 \)             | + (113-110-89+98)^2                                    |       |             |       |
| \( \ldots \)                  | + (113-110-89+98)^2                                    |       |             |       |
| \( pa_{14} = 12 \)            | + (73-81-89+98)^2                                      |       |             |       |
| \( \ldots \)                  | + (73-81-89+98)^2                                      |       |             |       |
| \( pa_{84} = 1 \)             | + (73-81-89+98)^2                                      |       |             |       |

| Total Variance | \( \Sigma (Y_{ij} - \bar{Y}_{**})^2 \) | AN - 1 | \( s^2 = S_t/(AN-1) \) | 145.68 |       |

where \( N \) = total number of subjects, \( A \) = number of groups for Factor A, \( \overline{Y}_{**} \) = the grand mean of \( Y \) across all measures, \( Y_{ij} \) = each individual score on \( Y \), and \( \overline{Y}_{p*} \) = the mean for each subject. \( \overline{Y}_{*A} \) = the mean for each measure.
### General Linear Model Approach

The model is given by:

\[ Y_{ij} = \mu_* + \alpha_j + \pi_i + \alpha\pi_i \]

<table>
<thead>
<tr>
<th>[ Y_{ij} = \bar{Y}_* ]</th>
<th>+ [ a_j ]</th>
<th>+ [ p_i ]</th>
<th>+ [ ap_i ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>127 = 98</td>
<td>+5</td>
<td>+12</td>
<td>+12</td>
</tr>
<tr>
<td>123 = 98</td>
<td>+5</td>
<td>+17</td>
<td>+3</td>
</tr>
<tr>
<td>111 = 98</td>
<td>+5</td>
<td>+14</td>
<td>−6</td>
</tr>
<tr>
<td>99 = 98</td>
<td>+5</td>
<td>+3</td>
<td>−7</td>
</tr>
<tr>
<td>99 = 98</td>
<td>+5</td>
<td>−7</td>
<td>+3</td>
</tr>
<tr>
<td>95 = 98</td>
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<td>−8</td>
<td>+6</td>
</tr>
<tr>
<td>87 = 98</td>
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<td>−2</td>
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<td>83 = 98</td>
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<td>−3</td>
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<td>103 = 98</td>
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<td>127 = 98</td>
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<td>+17</td>
<td>+7</td>
</tr>
<tr>
<td>127 = 98</td>
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<td>+14</td>
<td>+10</td>
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<td>+3</td>
<td>−11</td>
</tr>
<tr>
<td>95 = 98</td>
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<td>−7</td>
<td>−1</td>
</tr>
<tr>
<td>103 = 98</td>
<td>+5</td>
<td>−8</td>
<td>+8</td>
</tr>
<tr>
<td>87 = 98</td>
<td>+5</td>
<td>−14</td>
<td>−2</td>
</tr>
<tr>
<td>87 = 98</td>
<td>+5</td>
<td>−17</td>
<td>+1</td>
</tr>
<tr>
<td>97 = 98</td>
<td>−1</td>
<td>+12</td>
<td>−12</td>
</tr>
<tr>
<td>121 = 98</td>
<td>−1</td>
<td>+17</td>
<td>+7</td>
</tr>
<tr>
<td>97 = 98</td>
<td>−1</td>
<td>+14</td>
<td>−14</td>
</tr>
<tr>
<td>121 = 98</td>
<td>−1</td>
<td>+3</td>
<td>+21</td>
</tr>
<tr>
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<td>−1</td>
<td>−7</td>
<td>−1</td>
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<tr>
<td>89 = 98</td>
<td>−1</td>
<td>−8</td>
<td>+0</td>
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<tr>
<td>81 = 98</td>
<td>−1</td>
<td>−14</td>
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<td>113 = 98</td>
<td>−10</td>
<td>+12</td>
<td>+13</td>
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<tr>
<td>89 = 98</td>
<td>−10</td>
<td>+17</td>
<td>−16</td>
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<td>+11</td>
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<td>−10</td>
<td>+3</td>
<td>−2</td>
</tr>
<tr>
<td>81 = 98</td>
<td>−10</td>
<td>−7</td>
<td>+0</td>
</tr>
<tr>
<td>73 = 98</td>
<td>−10</td>
<td>−8</td>
<td>−7</td>
</tr>
<tr>
<td>81 = 98</td>
<td>−10</td>
<td>−14</td>
<td>+7</td>
</tr>
<tr>
<td>73 = 98</td>
<td>−10</td>
<td>−17</td>
<td>+2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{SS}_Y &= \Sigma \alpha^2 + \Sigma \pi^2 + \Sigma\pi\alpha^2 \\
\text{SS}_Y &= \Sigma (Y_{ij} - \mu_\ast)^2 = \Sigma (\mu_{\ast j} - \mu_\ast)^2 + \Sigma (Y_{ij} - \mu_{\ast j} - \mu_\ast)^2 \\
\text{SS}_Y &= \Sigma (Y_{ij} - \bar{Y}_\ast)^2 = \Sigma (\bar{Y}_{\ast j} - \bar{Y}_\ast)^2 + \Sigma (Y_{ij} - \bar{Y}_{\ast j} - \bar{Y}_\ast)^2 \\
\text{SS}_Y &= 4516 = (\text{SS}_A = 1056) = (\text{SS}_P = 1236) = (\text{SS}_{A\times P} = 2224)
\end{align*}
\]
**Reliability Example**

In the One-Way Repeated Measures ANOVA, two factors represent separate sources of variance. Their interaction presents an independent sources of error variation. Suppose a questionnaire with five Likert-type items with a 5-point scale.

<table>
<thead>
<tr>
<th>Person</th>
<th>Item 1 (A₁)</th>
<th>Item 2 (A₂)</th>
<th>Item 3 (A₃)</th>
<th>Item 4 (A₄)</th>
<th>Item 5 (A₅)</th>
<th>( \overline{y}_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>P₂</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>P₃</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>P₄</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>P₅</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>P₆</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<tr>
<td>P₇</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>4</td>
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<tr>
<td>P₈</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>P₉</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>P₁₀</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>P₁₁</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3.8</td>
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<tr>
<td>P₁₂</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>P₁₃</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>P₁₄</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>P₁₅</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

\( \overline{y}_1 = 3 \quad \overline{y}_*1 = 3 \quad \overline{y}_*2 = 3 \quad \overline{y}_*3 = 3 \quad \overline{y}_*4 = 2 \quad \overline{y}_*5 = 3 \quad \overline{y}_{**} = 2.8 \)

**ANOVA MODEL:** \( Y_{ijk} = \mu^{**} + \alpha_j + \pi_i + \pi\alpha_{ij} + \epsilon_{ij} \)

**SPSS Output from GLM**

<table>
<thead>
<tr>
<th>Effect</th>
<th>Value</th>
<th>F</th>
<th>Hypothesis df</th>
<th>Error df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA Pillai's Trace</td>
<td>.485</td>
<td>2.593&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.000</td>
<td>11.000</td>
<td>.095</td>
</tr>
<tr>
<td>Wilks’ Lambda</td>
<td>.515</td>
<td>2.593&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.000</td>
<td>11.000</td>
<td>.095</td>
</tr>
<tr>
<td>Hotelling’s Trace</td>
<td>.943</td>
<td>2.593&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.000</td>
<td>11.000</td>
<td>.095</td>
</tr>
<tr>
<td>Roy’s Largest Root</td>
<td>.943</td>
<td>2.593&lt;sup&gt;a&lt;/sup&gt;</td>
<td>4.000</td>
<td>11.000</td>
<td>.095</td>
</tr>
</tbody>
</table>

<sup>a</sup> Exact statistic
SPSS Output from GLM Measure: MEASURE_1

Mauchly's Test of Sphericity

<table>
<thead>
<tr>
<th>Within Subject Effect</th>
<th>Mauchly's W</th>
<th>Approx. Chi-Square</th>
<th>df</th>
<th>Sig.</th>
<th>Epsilon^b</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORA</td>
<td>.429</td>
<td>10.514</td>
<td>9</td>
<td>.315</td>
<td>.717</td>
</tr>
</tbody>
</table>

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
a. May be used to adjust the degrees of freedom of the averaged tests of significance.
Corrected tests are displayed in the Tests of Within-Subjects Effects table
b. Design: Intercept+FACTORA Within Subject Design: FACTORB

Residual SSCP Matrix

<table>
<thead>
<tr>
<th>Sum-of-Squares and Cross-Products</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>1.000</td>
<td>.429</td>
<td>1.000</td>
<td>.357</td>
<td>.286</td>
</tr>
<tr>
<td></td>
<td>.429</td>
<td>2.000</td>
<td>1.571</td>
<td>.143</td>
<td>1.357</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>1.571</td>
<td>4.000</td>
<td>.143</td>
<td>1.571</td>
</tr>
<tr>
<td></td>
<td>.357</td>
<td>.143</td>
<td>.143</td>
<td>1.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>.286</td>
<td>1.357</td>
<td>1.571</td>
<td>.000</td>
<td>2.000</td>
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</tbody>
</table>

Correlations

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>.303</td>
<td>.500</td>
<td>.357</td>
<td>.202</td>
</tr>
<tr>
<td>.303</td>
<td>1.000</td>
<td>.556</td>
<td>.101</td>
<td>.679</td>
</tr>
<tr>
<td>.500</td>
<td>.556</td>
<td>1.000</td>
<td>.071</td>
<td>.556</td>
</tr>
<tr>
<td>.357</td>
<td>.101</td>
<td>.071</td>
<td>1.000</td>
<td>.000</td>
</tr>
<tr>
<td>.202</td>
<td>.679</td>
<td>.556</td>
<td>.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

From Cronbach (1951), the Coefficient Alpha = \( A/(A-1)[1 - (\sum s_{ii}/\sum \sum s_{ij})] \).
where \( \sum s_{ii} = [(1+2+4+1+2)+((.429+1.00+.357+.286+1.571+.143+1.357+.143+1.571+.000)] = 23.714 \)
is the sum of all elements, \( \sum s_{ij} = (1+2+4+1+2) = 10 \) is the sum of the variances (the diagonal of the covariance matrix), and \( A \) = the number of items. Thus Alpha = \((5/4)(1-(10/23.714)) = 1.2(0.5783) = .7229.\)

From Cronbach (1951), the Standardized Coefficient Alpha = \( A(\bar{r})/[1+(A-1)\bar{r}] \),
where \( \bar{r} = [(0.303+.500+.357+.202+.556+.101+.679+.556+.000)/10] = .3325 \) is the average of the 10 correlations and \( A \) = the number of items. Thus Alpha = \((5(.3325))/[1+(4(.3325))] = .7135\)

**Calculation of Epsilon**

where \( s_{ij} \) is any element of the covariance matrix, \( \bar{s}_i = (1+2+4+1+2)/5 = 2 \) is the mean of variances, \( \bar{s}_{ii} = (23.714)/25 = 0.9486 \) is the mean of all elements, and \( \bar{s}_i \) is the mean of the \( i^{th} \) row of the covariance matrix. Thus, \( \hat{\varepsilon} = (A^2(\bar{s}_{ij} - \bar{s}_{ii})^2)/(A-1)(\sum \sum s_{ij}^2 - 2A \sum \bar{s}_{i}^2 + A^2 \bar{s}_{ii}^2) \)

\[ \varepsilon = \frac{A^2(\bar{s}_{ij} - \bar{s}_{ii})^2}{(A-1)(\sum \sum s_{ij}^2 - 2A \sum \bar{s}_{i}^2 + A^2 \bar{s}_{ii}^2)} \]

\[
\hat{\varepsilon} = \frac{((25)(2-0.9486))^2}{[(4)((42.42343)-((10)(5.52855)))+((25)(0.89977))]} = 0.717
\]
The use of ANOVA to estimate internal consistency reliability was first proposed by Hoyt (1941). The Hoyt reliability is identical to the Cronbach’s (1951) Alpha.

Hoyt reliability = 1 - (MS_{AxP}/MS_p) = Alpha = 1 - (1.314/4.743) = .7229

This formulation is based on the perspective that MS_{AxP} is measurement error or error due to inconsistency. In the calculation of SS_{AxP}, note that for each individual score the Person effect \( \bar{Y}_{x} - \bar{Y}_{y} \) and the Item effect \( \bar{Y}_{p} - \bar{Y}_{y} \) are subtracted, yielding \( \bar{Y}_{ij} - \bar{Y}_{p} - \bar{Y}_{y} + \bar{Y}_{y} \). If an individual’s score on any item (\( Y_{ij} \)) can be perfectly predicted from the combination of how well they did on the test in total (i.e., the person’s mean, \( \bar{Y}_{p} \)) and how “difficult” the item was (i.e., the item mean, \( \bar{Y}_{y} \)), then the residual (i.e., \( Y_{ij} - \bar{Y}_{p} - \bar{Y}_{y} + \bar{Y}_{y} \)) is 0 and that response is consistent. If an individual’s scores on any item can not be perfectly predicted from the combination of how well they did on the test in total (i.e., the person’s mean, ) and item “difficulty,” then there is inconsistency. Squaring these residuals provides an estimate variance due to inconsistency or measurement error.

It should also be noted that the Person variance in the denominator is based on MS_p. There is no guarantee that this will be larger than MS_{AxP}. Just as the expected value of an F-ratio is 1 but can be less than 1 in any sample, the ratio of (MS_{AxP}/MS_p) is expected to be 1 under conditions of inconsistency. However, imagine that some students take an exam and respond very differently to the items, however, they all end up with almost identical total scores. This means the items did not contribute to separating these students on their ability (i.e., item discrimination). In this scenario the person variance would be small, but the error variance would be relatively large. Cronbach’s Alpha would be negative. The concept of negative reliability is a oxymoron, you cannot have negative variance. However, there is nothing in any of the formulas to keep the reliability estimate from being slightly negative. A negative Cronbach’s Alpha usually comes about because (1) the sample of test takers had homogeneous test scores or (2) because a few items had negative correlations (i.e., negative discrimination coefficients) with the other items. The second scenario often occurs when a Likert-type item is a “reversal” item (i.e., stated with negative framing), but the data analysts forgets to recode the variable.
Output from SPSS Scale - Reliability Analysis (Alpha Model)

***** Method 2 (covariance matrix) will be used for this analysis *****

RELIABILITY ANALYSIS - SCALE (ALPHA)

N of Cases = 15.0

N of Statistics for Mean Variance Std Dev Variables
Scale 14.0000 23.7143 4.8697 5

Item Means
Mean Minimum Maximum Range Max/Min Variance
2.8000 2.0000 3.0000 1.0000 1.5000 .2000

Item Variances
Mean Minimum Maximum Range Max/Min Variance
2.0000 1.0000 4.0000 3.0000 4.0000 1.5000

Inter-item Covariances
Mean Minimum Maximum Range Max/Min Variance
.6857 .0000 1.5714 1.5714 1.000E+20 .3697

Inter-item Correlations
Mean Minimum Maximum Range Max/Min Variance
.3324 .0000 .6786 .6786 1.000E+20 .0524

Item-total Statistics

<table>
<thead>
<tr>
<th>Scale</th>
<th>Mean</th>
<th>Variance</th>
<th>Std Dev</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>if Item</td>
<td>if Item</td>
<td>Total</td>
<td>Multiple</td>
<td>if Item</td>
</tr>
<tr>
<td>Deleted</td>
<td>Deleted</td>
<td>Correlation</td>
<td>Correlation</td>
<td>Deleted</td>
</tr>
<tr>
<td>A1</td>
<td>11.0000</td>
<td>18.5714</td>
<td>.4807</td>
<td>.3628</td>
</tr>
<tr>
<td>A2</td>
<td>11.0000</td>
<td>14.7143</td>
<td>.6452</td>
<td>.5159</td>
</tr>
<tr>
<td>A3</td>
<td>11.0000</td>
<td>11.1429</td>
<td>.6419</td>
<td>.4974</td>
</tr>
<tr>
<td>A4</td>
<td>12.0000</td>
<td>21.4286</td>
<td>.1389</td>
<td>.1490</td>
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<td>A5</td>
<td>11.0000</td>
<td>15.2857</td>
<td>.5813</td>
<td>.5186</td>
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</table>

Analysis of Variance

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Sq.</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between People</td>
<td>66.4000</td>
<td>14</td>
<td>4.7429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within People</td>
<td>85.6000</td>
<td>60</td>
<td>1.4267</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Measures</td>
<td>12.0000</td>
<td>4</td>
<td>3.0000</td>
<td>2.2826</td>
<td>.0717</td>
</tr>
<tr>
<td>Residual</td>
<td>73.6000</td>
<td>56</td>
<td>1.3143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>152.0000</td>
<td>74</td>
<td>2.0541</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Mean</td>
<td>2.8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Intraclass Correlation Coefficient

Two-Way Mixed Effect Model (Consistency Definition):
People Effect Random, Measure Effect Fixed
Single Measure Intraclass Correlation = .3429*

95.00% C.I.: Lower = .1244 Upper = .6287
F = 3.6087 DF = ( 14, 56.0) Sig. = .0003 (Test Value = .0000)
Average Measure Intraclass Correlation = .7229**

95.00% C.I.: Lower = .4154 Upper = .8943
F = 3.6087 DF = ( 14, 56.0) Sig. = .0003 (Test Value = .0000)
*: Notice that the same estimator is used whether the interaction effect is present or not.
**: This estimate is computed if the interaction effect is absent, otherwise ICC is not estimable.

Reliability Coefficients 5 items

Alpha = .7229 Standardized item alpha = .7135
### Computations

Within-Subjects ANOVA Source Table:  

\[ Y_{ijk} = \mu + \alpha_j + \pi_i + \pi\alpha_{ij} + \epsilon_{ij} \]

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BETWEEN-SUBJECTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor P  ((\sum \pi^2_j))</td>
<td>(\sum A(\bar{Y}<em>{p*} - \bar{Y}</em>{**})^2)</td>
<td>(N - 1)</td>
<td>(\frac{SS_p}{(N-1)})</td>
<td></td>
</tr>
<tr>
<td>(p_1 = 1.2)</td>
<td>((4 - 2.8)^2)</td>
<td>(15)</td>
<td>(66.40)</td>
<td></td>
</tr>
<tr>
<td>(p_2 = 1.2)</td>
<td>(+ (4 - 2.8)^2)</td>
<td>(14)</td>
<td>(4.743)</td>
<td></td>
</tr>
<tr>
<td>(p_3 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(5)</td>
<td>(3.0)</td>
<td></td>
</tr>
<tr>
<td>(p_4 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(12)</td>
<td>(3.324)</td>
<td></td>
</tr>
<tr>
<td>(p_5 = 1.2)</td>
<td>(+ (4 - 2.8)^2)</td>
<td>(4)</td>
<td>(4.0)</td>
<td></td>
</tr>
<tr>
<td>(p_6 = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(5)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_7 = 1.2)</td>
<td>(+ (4 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_8 = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_9 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{10} = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{11} = 1.0)</td>
<td>(+ (3.8 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{12} = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{13} = -1.6)</td>
<td>(+ (1.2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{14} = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(p_{15} = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(56)</td>
<td>(1.314)</td>
<td></td>
</tr>
<tr>
<td>(= 66.40)</td>
<td>(= 66.40)</td>
<td>(= 66.40)</td>
<td>(= 66.40)</td>
<td></td>
</tr>
<tr>
<td><strong>WITHIN-SUBJECTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FACTOR A  ((\sum \alpha^2_k))</td>
<td>(\sum n(\bar{Y}<em>{*A} - \bar{Y}</em>{**})^2)</td>
<td>(A - 1)</td>
<td>(\frac{SS_A}{(A-1)})</td>
<td>(\frac{MS_A}{MS_{AxP}})</td>
</tr>
<tr>
<td>(a_1 = 0.2)</td>
<td>((3 - 2.8)^2)</td>
<td>(5)</td>
<td>(4)</td>
<td>(3.0)</td>
</tr>
<tr>
<td>(a_2 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(4)</td>
<td>(12)</td>
<td>(3.324)</td>
</tr>
<tr>
<td>(a_3 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(4)</td>
<td>(12)</td>
<td>(3.324)</td>
</tr>
<tr>
<td>(a_4 = -0.8)</td>
<td>(+ (2 - 2.8)^2)</td>
<td>(4)</td>
<td>(12)</td>
<td>(3.324)</td>
</tr>
<tr>
<td>(a_5 = 0.2)</td>
<td>(+ (3 - 2.8)^2)</td>
<td>(4)</td>
<td>(12)</td>
<td>(3.324)</td>
</tr>
<tr>
<td>(= 12)</td>
<td>(= 12)</td>
<td>(= 12)</td>
<td>(= 12)</td>
<td></td>
</tr>
<tr>
<td>Within-Subjects (Error)  ((\sum \pi\alpha_{ij}))</td>
<td>(\sum (Y_{ij} - \bar{Y}<em>{p*} - \bar{Y}</em>{*A} + \bar{Y}_{**})^2)</td>
<td>((A-1)(N-1))</td>
<td>(\frac{SS_{AxP}}{df_{AxP}})</td>
<td>(\frac{MS_{AxP}/MS_{AxP}}{1.314})</td>
</tr>
<tr>
<td>(p_{11}a_1 = -0.2)</td>
<td>((4 - 4 - 3.2 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(p_{21}a_1 = -0.2)</td>
<td>(+ (4 - 4 - 3.2 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(p_{12}a_2 = -0.2)</td>
<td>((4 - 4 - 3.2 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(p_{22}a_2 = -1.2)</td>
<td>(+ (4 - 4 - 3.2 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(p_{15}a_5 = -0.2)</td>
<td>((1 - 2 - 3 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(\cdots)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(p_{15,4}a_5 = -1.2)</td>
<td>((1 - 2 - 3 + 2.8)^2)</td>
<td>(5)</td>
<td>(56)</td>
<td>(1.314)</td>
</tr>
<tr>
<td>(= 73.60)</td>
<td>(= 73.60)</td>
<td>(= 73.60)</td>
<td>(= 73.60)</td>
<td></td>
</tr>
<tr>
<td>Total Variance</td>
<td>(\sum (Y_{ij} - \bar{Y}<em>{i} - \bar{Y}</em>{A})^2)</td>
<td>(AN - 1)</td>
<td>(s^2 = \frac{S^2}{(AN-1)})</td>
<td>(= 2.054)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 152)</td>
<td>(= 2.054)</td>
<td>(= 2.054)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 74)</td>
<td>(= 2.054)</td>
<td>(= 2.054)</td>
</tr>
</tbody>
</table>

Where \(N = \) total number of subjects, \(A = \) number of groups for Factor A, \(\bar{Y}_{**} = \) the grand mean of \(Y\) across all measures, \(Y_{ij} = \) each individual score on \(Y\), and \(\bar{Y}_{p*} = \) the mean for each subject. \(\bar{Y}_{*A} = \) the mean for each measure.